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PROCEEDINGS
OF THE
AMERICAN SOCIETY OF MICROSCOPISTS

TENTH ANNUAL MEETING.

ANNUAL ADDRESS OF THE PRESIDENT:

THE MICROSCOPE AS A FACTOR IN A STUDY OF THE BEHAVIOR OF METALS UNDER VARIATIONS OF TEMPERATURE.

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Microscopy is a cosmopolitan science. We may go farther than this and say that microscopy is more nearly cosmopolitan in its character than any other science. If I did not believe this to be true, I should not have consented to occupy the honorable position which I now hold by your suffrages, for there are many members of this society to whom the honor more justly belongs by virtue of greater familiarity with the technics of our science. I suppose that I am indebted to this expression of your confidence on account of the use which I have made of the microscope as an essential factor in a single line of research.

It is the glory of our science that the microscope supplements the natural vision to such an extent that we can submit nearly every theory, nearly every deduction from experiment, nearly every fact of observation, to the supreme and only test by which a real truth in nature can be established, viz.: through the medium of one of the senses with which we have been endowed by the Creator. It has been said that microscopy has no claim to be regarded as a science, and that the microscope is simply an instrumental agent occupying, with respect to other sciences, a position similar to that which the telescope sustains in its relation to astronomy. A convincing answer to this criticism is found in the fact that the telescope is limited in its application to a comparatively narrow field of research. Where the telescope answers a single question the microscope answers a thousand. Spectroscopy has become a recognized science, not so much because of its revelations in regard to the nature of light, as on account of the application of the spectroscope as an instrument, to the study of the physical properties of matter and of motion not only on the earth but in worlds other than our own.

In discussing the question whether microscopy can be regarded as a science, we must always bear in mind the fact that a science is only a convenient name *for a group of similar laws of nature*, and that the term is properly applicable not only to the development of these laws, but to their application to the useful economies of life. Thus we have the science of engineering, in which mathematical analysis is as much an essential part as skill in mechanical construction. But this analysis would serve no useful purpose if it did not rest ultimately on facts of observation.

The limitations which necessarily belong to a definition of physical science are clearly expressed by Tate in his most admirable treatise on Heat. He says: "Nothing can be learned as to the physical world save by observation and experiment, *or by mathematical deductions from data so obtained*." Now the microscope as an instrument of research stands unrivaled, not only in respect to the precision of the observations made with its aid, but also in the universality of its application in furnishing what Tate calls "the data so obtained."

Each succeeding year witnesses an extension of the range of its applications. Within a few years, while retaining its claim as an essential factor in scientific research, it has also become a very material aid in many mechanical industries. It is a common impression that the microscope is too delicate an instrument to be used in the ordinary operations of mechanical construction, and that the apparent necessity of using transmitted light for the purpose of illumination is an absolute barrier to any extended employment of the instrument. The latter difficulty is entirely obviated by the use of the opaque illuminator, invented by Tolles, by which a bright metal surface can be examined with the utmost ease, while actual experience has shown that it is by no means necessary that the instrument should be mounted upon massive piers insulated from surrounding objects.

I cannot more forcibly combat this impression than by referring to two cases within my own experience. The Proceedings of the Society of Mechanical Engineers for 1884, contains a description of a method of cutting a screw in which each thread is made to correspond in pitch with equal subdivisions of a standard yard traced upon a metal bar. The screw for the engine constructed for the Cornell University was made in this manner. Professor Anthony has shown that the maximum accumulated error of the screw does not reach two mikrons for a limit of twenty inches, while the actual error at any selected point does not reach one mikron. This screw was cut in the manner indicated, in the third story of a building occupied by machinery which produced a decided tremor in every room. It was only found necessary to make the attachment of the microscope to the compound rest of the lathe very firm, and to brace the bed of the lathe very securely from the floor.

The writer was recently called upon to "level up" the bed of a very heavy planer, having ways eighteen feet in length. Several days had been spent in securing as good an adjustment as could be obtained with the aid of a spirit level of special construction. A plank twenty-two feet in length, eight inches in width, and two inches in thickness, was set up edgewise beside the platen of the planer, but insulated from it. A groove half an inch wide and half an inch deep was ploughed in the upper face of the plank, and after having stopped both ends, the groove was filled with mercury. The surface of the mercury then formed an invariable plane of reference. The microscope was securely attached to the platen, and adjusted for a sharp focus upon the surface of the mercury at one end. The platen was then moved along until the microscope occupied a position near the other end of the groove. This end was then adjusted by elevation or depression, as required, until the surface of the mercury was sharply in focus. After two trials it was found that the surface of the mercury was at the same constant focal distance from the microscope as indicated by the sharpness of definition. Notwithstanding the fact that extreme care had been taken in the original adjustment by the aid of the spirit level, it was found that as the platen moved toward the central part of the bed, the focus became more and more indistinct, indicating that the central part was too low. The proper elevation was then made at these points by means of heavy set-screws, when it was found that the mercury was sharply in focus under the objective throughout the entire range of motion. As a check upon the accuracy of the adjustment a surface plate, eight feet in length, was now planed, when it was found that the deviation from a true surface did not at any point, exceed the third part of the thickness of tissue paper. Two facts of considerable importance are to be noticed in connection with this experiment. First, that the time occupied for the complete adjustment was only twenty-five minutes; and, second, that during the entire operation the machinery of the shop was running at half speed.

These and similar observations have led the writer to advocate a more extended use of the microscope in the every-day work of the machine shop. By attaching the microscope firmly to the slide rest of the lathe, the ordinary operations of turning shoulders to a given length, and of cylinders to a given diameter can be more expeditiously, more exactly and more economically performed than by the usual method.

It is freely admitted by mechanicians that a decided advance in mechanical constuction would be made by the employment of uniform measures of length. This can be easily and profitably accomplished in any well regulated shop, employing as many as fifty hands, by delivering from a standards-room any desired unit of length, in the same way that tools are delivered from a tool-room. The expense of a comparator, from which any measure of length could be obtained within a limit of time which would not ordinarily exceed one minute, would not be great. If this comparator were placed in charge of a person familiar with its use, and in a convenient location, any workman could have a calliper set for him in half the time that would be required in setting it to a scale by the usual method; the precision would be incomparably greater, and absolute uniformity would be secured in every dimension of length employed. The various points to which I have briefly called attention, are to be considered simply as illustrations of the many ways in which the useful service of the microscope may be extended.

In the address which I am called upon to make this evening, as President of the American Society of Microscopists, I have selected a single application of the microscope in scientific research. *I beg to call your attention to the microscope as a factor in a study of the behavior of metals under variations of temperature.*

If a bar of metal which has the faces of each end parallel and at right angles to its axis, is submerged in melting ice, the perpendicular distance between the two faces may be said to represent a definite unit of length at the temperature of 32° F. or 0° C. If this distance is identical in length under similar conditions, with a certain bar of platinum now deposited at the International Bureau of weights and measures at Breteuil, near Paris, and designated the "*Metre des Archives*," the length of the bar is said to be one meter. If now the bar is submerged in a liquid which has throughout its entire mass a temperature one degree higher than that of melting ice, its length, after it has reached the same temperature as the liquid, will be increased by a certain fraction of its entire length. If this length is subdivided into one million equal parts, and if the increase is, for example, ten parts in one million, the coefficient of expansion of the metal is said to be ten mikrons. If the increase in length proceeds uniformly for each and every increment of temperature, we can say, for example, that the length of the bar at 100° C. will be 1000 mikrons, or one millimeter greater than it was at 0° C. We can also say that if the temperature of the entire mass of metal is again reduced to 0°, the length of the bar will be exactly the same as it was before the increase of temperature took place.

There is some evidence that when certain metals are exposed to very violent changes in temperature, as when zinc is removed from a temperature of 100° C. and is submerged in melting ice, the molecular arrangement of the metal is disturbed to such an extent that the return to its original condition may be delayed for several days, and even for several weeks; but it cannot, at the present time, be possitively asserted that the return will not ultimately take place.

It will be noticed that the definition of the coefficient of expansion which has been given, viz: the increase in length due to an increase in temperature from 0° to 1°, contains the important limitation that the entire mass of the metal shall have reached the temperature of 0°

If the total expansion due to a rise from the temperature v to the temperature v^1 is defined to be the increase in length corresponding to a rise of 1° multiplied by the number of degress between v and v^1, we must understand that the change for a single degree is at every point a constant quantity.

Let a=the length of a meter at 0° C.

v=the observed temperature of the entire mass of the metal at any time, T.

b=the change in length corresponding to a change in temperature from 0° to 1°

Δ=the length of the meter at the temperature v.

We shall then have—

$$\Delta = a + bv \qquad (1)$$

Every observed measure of length will finish an equation of the form (1).

For any other temperature b^1, the length Δ^1 will be expressed by the equation,

$$\Delta^1 = a + bv^1 \qquad (2)$$

Since a and b relate by definition only to the temperature 0°, they are therefore constant.

It will be seen that the difference in length due to a change from v to v^1 will be found by subtracting equation (1) from equation (2)

or: $$\Delta^1 - \Delta = (v^1 - v)b$$

whence $$b = \frac{\Delta^1 - \Delta}{v^1 - v} \qquad (3)$$

In order that equations (1) and (2) shall hold generally, it is necessary that the coefficient of expansion b shall be constant for the entire range of temperature between v and v^1. In other words, if the increase in length is 10 mikrons from 0° to 1°, the increase from 1° to 2° must also be 10 mikrons, in order that we may be able to say that the increase from 0° to 2° is equal to 2x10 mikrons.

It is just at this point that our inquiry begins. With few exceptions it is assumed by physicists *that the coefficient of expansion increases in value with the temperature.*

Our equation (1) would in this case bccome

$$\Delta = a + bv + \tfrac{1}{2}b^1v \qquad (4)$$

in which b^1 represents the correction to b required at the mean temperature $\frac{o+v}{2}$ or $\frac{1}{2}v$. Now if the law of expansion is not a direct and linear function of the temperature, equation (4) must be regarded as only an approximation to the true law, since we have assumed that b^1 is a constant for the mean temperature $\frac{1}{2}v$. Theoretically, we should have:

$$\varDelta = a + bv + \tfrac{1}{2}b^1v + \tfrac{3}{4}b^2v + \tfrac{7}{8}b^3v + \tfrac{15}{16}b^4v + \&c. \quad (5)$$

in which the coefficients b^2 b^3 and b^4 hold for the temperatures $\frac{\frac{1}{2}v+v}{2}$, $\frac{\frac{3}{4}v+v}{2}$, $\frac{\frac{7}{8}v+v}{2}$, &c.

It might at first sight seem to be purely a mathematical excursion to determine whether the law of expansion is expressed by equation (1) or by equation (5); but if it is determined by equation (1) we can make use of this fact in an invesitgation of the highest practical importance, while if the coefficient of expansion is not a constant for each and for every degree of temperature considered, we become hopelesly involved in the determination of the coefficients b, b^1, b^2, b^3, b^4, &c. from a series of equations of the form (5).

Let us state as clearly as possible the problem which we have before us.

The temperature is indicated by a thermometer. If we employ a mercurial thermometer to measure the temperature, the hight of a column of mercury is indicated by a zero graduation when the temperature of the entire mass of the mercury in the bulb of the thermometer and of the glass of which the stem is made, is the same as that of melting ice. The hight of the column at the boiling point of water is marked 100° C. under similar conditions. We can now say that the readings of the scale truly represent the temperature at these two specific points. If the distance between these two points is subdivided into an equal number of parts, for example, 100, we can say of any reading of the hight of the mercury column at intermediate points, that this scale-reading represents the real temperature if two conditions are fulfilled; first, if the mercury expands uniformly for each degree of temperature, and second, if the bore of the glass tube which contains the mercury has the same diameter at every point. Since the latter condition can never be exactly fulfilled in the process of blowing the glass except by accident, and since the first condition introduces an additional element of doubt on account of the probable increase of the viscosity of the mercury as it approaches its freezing point, considerable doubt must always attach to the reading of a mercurial thermometer, especially at very low temperatures, even after a large part of the errors have been removed by the process of calibration. It seems to be fairly well established, that for high temperatures mercurial thermometers read too high when compared with the air thermometer, which is generally regarded as reliable in its indications. In a case cited by Sir William Thompson, the readings of three mercurial thermometers were 329.8°, 327.2°, 321.8°, respectively, corresponding to 320.0° for the air thermometer. All of the investigations made by the writer point to the conclusion that mercurial thermometers as a class, read too low when the temperature falls below 10° to 15° Fahr.

It should be said, however, that recent comparisons of several mercurial thermometers with an air thermometer at temperatures as low as minus 38° Fahr. made by the U. S. Signal Office, indicate a very close agreement. The correction of the mercurial standard Green No. 1503 at minus 28° was found to be only —0°.1. The observations of Grunmach, made in 1881, show a variation of less than 0°.1 at —20° C.

Under this state of uncertainty, it will be admitted that any independent method of measuring the temperature in which there shall be an entire freedom from the class of errors peculiar to mercurial thermometers, and to the air thermometer, would be of great value in removing existing doubts.

The method of indicating temperature by noting the relative changes in length between two bars of metal having widely different coefficients of expansion, has received considerable attention. It has been used in several trigonometrical and topographical surveys, but with results not altogether satisfactory. In addition to the assumed uncertainty with regard to the constancy of the coefficients of expansion of the two metals employed, an additional element of doubt has been introduced by joining the metals by a ligature near the center, thus introducing a compound effect due to the different rates at which the two bars arrived at complete saturation after changes of temperature occur.

The present investigation is an attempt to show that the relative expansions of Baily's metal, of Jessup's steel, and of Chance & Son's glass, are linear functions of the temperature, within the limits of the accidental errors of observation, and that these metals may therefore be safely employed to measure variations of temperature at least between $-5°$ C. and $+35°$ C. These measures will therefore serve as a check upon the corresponding readings of mercurial thermometers. The following example will serve as an illustration of the practical bearing of this inquiry.

It will be shown subsequently that the bronze bar of Baily's metal expands 6.20 divisions of the micrometer for each degree Fahrenheit in temperature more than the bar of Jessup's steel under the same conditions. At 62.0 degrees Fahrenheit the yard upon the steel was found to be 9.8 divisions longer than the yard upon the bronze bar. At minus 19.8 degrees the steel yard was found to be 490.5 divisions longer than the bronze yard. Hence by equation (1)

-490.5 div. $= -9.8 + [62.0 + 19.8] \times -6.20 = -517.0$ divisions.

The observed difference is 490.5 divisions while the difference due to an increase of 6.20 divisions for each degree of temperature is 517.0 divisions. Admitting that the observed value is correctly determined, and that the change for each degree is a real constant, there is but one way of reducing the relation 517.0 divisions to 490.5 divisions, viz., by reducing the temperature reading 4.26°. The thermometer reading at this point was therefore 4.26° too low. In a similar manner we may obtain the corrections of any mercurial thermometer at all points other than 0° and 100° on the centigrade scale and 32° and 212° on the Fahrenheit scale.

The particular bars of metal described have been chosen on account of the great number of comparisons between the relative lengths of the units traced upon them, which have been made since the year 1881. The results obtained previous to the present series of observations will be found recorded in the Proceedings of the American Academy of Arts and Sciences, Vol. XVIII, pages 287–398, and in the Proceedings of this Society for the year 1885. The agreement in the values of the various determinations of the relative coefficients of expansion of these metals are satisfactory in confirming the conclusions reached in this paper, and in establishing the fact that the relative coefficients have not undergone any change since 1881.

The bronze bar, which is designated R_2, has the same composition and cross section as the Imperial yard of Great Britain. It is composed of 16 parts of copper, 2½ parts of tin, and 1 part of zinc. Both the yard and the meter are traced upon platinum-iridium plugs securely inserted in the bar. Each unit of length is nearly standard at 62.0° Fahrenheit. The present investigation relates to the yard.

The yard and meter upon steel are traced upon polished steel plugs inserted in a bar one and a quarter inches in depth and three quarters of an inch wide. The material is the ordinary commercial steel as sold by the makers. This bar is designated R_3.

The glass bar designated G has the dimensions 41×1¾×1¾ inches. This bar is the property of the Standards Department of the British Board of Trade. It was made by Chance & Sons in 1870, upon a special order from the Department.

Another important limitation must be noted in this discussion. The results obtained all depend upon what are termed air contacts with the metals: that is, the bars of metal were supported upon the comparator at their neutral points in such a manner as to allow a free circulation of air at every point of their surfaces. The thermometers employed were placed upon the upper surfaces of the bars with a slight elevation of the ends farthest from the bulbs. After the bars had been adjusted for position side by side at the commencement of the series of observations, their positions remained undisturbed throughout the entire series. The stops of the comparator having been set at a distance apart approximately equal to the length of one yard, the length of the yard on each bar was compared with the distance between the stops. A single comparison usually occupied about five minutes, and during this time the constancy of the distance between the stops could be safely assumed, since this distance could only vary with the movement of the large mass of metal in the bed-plate of the comparator. By setting the microscope for coincidence with the initial line of each standard in succession, the comparisons for length were made without touching either of the bars. The nearest point of approach of the person of the observer to the bars was about 6 inches.

This explicit statement with regard to the manner in which the comparisons were made, and of the conditions under which the temperatures were obtained, is made in order that there may be no doubt concerning the basis upon which the conclusions reached in this paper rest.

It is stoutly affirmed in certain quarters that the method of comparison under air contacts is far less reliable than comparisons when both the bars compared and the thermometers employed to measure the temperature are submerged in a liquid. Even if the correctness of the method of air contact is admitted, it is insisted that the temperature should be measured by inserting the thermometers in wells sunk in the bars of metal and filled with some liquid, preferably mercury.

We have here three distinct problems, each one of the most difficult and intricate character.

First. When given masses of water, of alcohol, of mercury, or of glycerine are removed from a given temperature, and are submitted to contact with the air at a different and a practically constant temperature, in how long time will they reach the temperature of the surrounding air? If the time in which a thermometer will come to rest under similar conditions is less than this observed time, to what extent can a mercurial thermometer, when submerged in a liquid, indicate the real temperature of the liquid when both are undergoing unknown variations of temperature. This inquiry does *not* relate to the comparison of different thermometers by submergence in the same liquid. It *does* relate to the indication of the real temperature of the liquid mass. Any number of thermometers may give true *relative* indications of temperature when subjected to a common condition, and yet *all* may give erroneous indications of the real temperature of the liquid mass with which they are surrounded. This inquiry will also properly include a determination of the laws under which the different liquids reach an equilibrium of temperature with the surrounding air, especially in connection with the upward and downward flow of thermal currents, which are well known to exist in different parts of a given liquid mass.

Second. Under what circumstances and conditions will the mercurial thermometer indicate the temperature of the material with which it is placed in contact, and are the changes which take place in the indications given by thermometers under variations of temperature due to causes which relate to the thermometer itself, or to the changes of temperature in the material with which they are in contact. This inquiry will include a discussion of the time required for thermometers to pass from any constant temperature to another temperature also constant, under the three conditions, air contact, liquid contact, and surface contact with metals of different masses.

Third. When metals of different masses are subjected to contact with liquids, in how long time will they reach the temperature of the entire mass of the surrounding liquid, and is this time greater or less than the time required for equilibrium to take place when the metals are exposed to contact with the air under the same variations of temperature. Under this head will be included the determination of the time required for different masses of metal to reach the stationary point after the temperature of the surrounding air has become practically constant; and also the length of time which elapses after a change of temperature in the air has taken place, before the entire effect of this change is seen in a corresponding change in length.

It will be found advantageous to introduce this discussion with a few fundamental propositions relating to heat, and especially those which have a special bearing upon the special problem under consideration.

Fourier, in his classical treatise on the Analytical Theory of Heat, has given in simple language the fundamental conceptions relating to heat, which have formed the basis of all subsequent investigations. We quote such of these as are pertinent to our present inquiry:—

(a) " Heat, like gravity, penetrates every substance in the universe, its rays occupy all parts of space. The object of our work is to set the mathematical laws which this element obeys."

(b) Observation has shown conclusively that different bodies possess in different degrees:—

(1) The power to contain heat.

(2) The power to receive or to transmit heat across their surfaces.

(3) The power to conduct heat throught the interior of their masses.

(c) When heat is unequally distributed among the different parts of a solid mass, it tends to attain equilibrium, and pass slowly from the parts which are more heated to the parts which are less heated, and at the same time it is dissipated at the surface and lost in the surrounding medium. The tendency to uniform distribution, and the spontaneous emission which acts at the surface of bodies continually, changes their temperature at different points. The problem of the propagation of heat consists in determining what is the temperature at each point of a body at a given instant, supposing that the initial temperatures are known.

(d) If we place a solid homogeneous mass in the form of a sphere or a cube in a medium maintained at a constant temperature, and if it remains immersed for a long time, it will acquire at all of its points a temperature differing very little from that of the fluid. Suppose the mass to be withdrawn in order to transfer it to a cooler medium; heat will begin to be dissipated at its surface; the temperature at different points of the mass will not be sensibly the same, and if we suppose it divided into an infinite number of layers by surfaces parallel to its external surface, each of these layers will transmit at each instant a certain quantity of heat to the layer which surrounds it. If it be imagined that each molecule carries a separate thermometer which indicates its temperature at every instant, the state of the solid will from time to time be represented by the variable system of all these thermometric hights. It is required to express the successive states by analytical formula so that we may know at any given instant the temperature indicated by each thermometer, and compare the quantities of heat which flow during the same instant, between two adjacent layers or into the surrounding medium.

(e) Let us examine also the case in which a rectangular prism of sufficiently great thickness, and of infinite length, being submitted at its extremity to a constant temperature, while the air which surrounds it is maintained at a less temperature, has at last arrived at a fixed state which it is required to determine. All of the points of the extreme section at the base of the prism have by hypothesis a common and permanent temperature. It is not the same with a section distant from the source of heat; each of the points of this rectangular surface parallel to the base has acquired a fixed temperature, but this is not the same at different points of the same section, and must be less at points nearer to the surface exposed to the air. We see also that at each instant there flows across a given section a certain quantity of heat, which always remains the same since the state of the solid has become constant. The problem consists in determining the permanent temperature at any given point of the solid, and the whole quantity of heat which, in a definite time, flows across a cross section whose position is given.

(f) The solution of these problems has made us understand that the effects of the propagation of heat depend in the case of every solid substance on three elementary quantities, which are

(1) Its capacity for heat.
(2) Its own or interior conductivity.
(3) Its exterior conductivity.

(g) It has been observed, that if two bodies of the same volume, and of different nature, have equal temperatures, and if the same quantity of heat be added to them, the increments of temperature are not the same; the ratio of their increments is the ratio of their capacities for heat. . . . The proper or interior conductivity of a body expresses the facility with which heat is propagated in passing from one internal molecule to another. The external or surface conductivity of a solid body depends on the facility with which heat penetrates the surface, and passes from this body into a given medium, or passes from this medium into a solid. The last property is modified by the more or less polished state of the surface; it varies also according to the medium in which the body is immersed, but the interior conductivity can only change with the nature of the solid.

(h) Radiant heat, which escapes from the surface of all bodies, and traverses elastic media or even spaces void of air, and occurring with widely varied phenomena, follows special laws. To complete our theory it was necessary to examine the laws which radiant heat follows on leaving the surface of a body. . . . It results from the observations of many physicists, and from our own experiments, that the intensity of the different rays which escape in all directions from any point on the surface of a heated body, depends on the angle which their directions make with the surface at the same point. We have found that the intensity of a ray diminishes as the ray makes a smaller angle with the element of the surface, and that it is in proportion with the sine of the angle. . . . Up to this point omission has been made of the power which all surfaces have of reflecting part of the rays which are sent to them. If this property were disregarded we should have only a very incomplete idea of the equilibrium of radiant heat. . . . Thus, the reflection of heat does not disturb the equilibrium of temperature, and does not introduce, while the equilibrium exists, any change in the laws according to which the intensity of the rays which leave the same point decreases proportionally to the the size of angle of emission.

(i) Different *quantities* of heat are measured by determining how many times they contain a fixed quantity, which is taken as the unit. Suppose a mass of ice having a definite weight (1 kilogramme) to be at the temperature 0°, and to be converted into water at the same temperature 0 degrees by the addition of a certain quantity of heat. The quantity of heat thus added is taken as the unit of measure. . . . To raise a kilogramme of iron, for example, from the temperature 0° to the temperature 1°, a new quantity of heat must be added to that which is already contained in the mass. The number C which denotes this additional quantity of heat is the specific capacity of iron for heat; the number C has very different values for different substances.

(j) We take as the measure of external conductivity of a solid body a coefficient h which denotes the quantity of heat which would pass in a definite time (a minute) from the surface of this body into atmospheric air, supposing that the surface had a definite extent (a square meter), that the constant temperature of the body was 1°, and that of the air 0°, and that the heated surface was exposed to a current of air of a given invariable velocity. This value of h is determined by observation. *The quantity of heat expressed by the coefficient is composed of two distinct parts, which cannot be measured except by very exact experiments, one is the heat communicated by way of contact to the surrounding air, the other much less than the first, is the radiant heat emitted.*

(k) All bodies have the property of emitting heat through their surface; the hotter they are the more they emit; the intensity of the emitted rays changes considerably with the state of the surface. . . . Every surface which receives heat from surrounding bodies reflects part and admits the rest; the heat which is not reflected but introduced through the surface, accumulates within the solid; and so long as it exceeds the quantity dissipated by irradiation the temperature rises. . . . The rays which tend to go out of heated bodies are arrested at the surface by a force which reflects part of them into the interior of the mass. The cause which hinders the incident rays from traversing the surface and which divides these rays into two parts, of which one is reflected and the other admitted, acts in the same manner as the rays which are diverted from the interior of the body towards external space. . . . The incident rays introduced into the mass and the rays emitted from the surface are equally diminished in quantity.

(l) Liquid substances possess also the property of transmitting heat from molecule to molecule, and the numerical value of their conductivity varies according to the nature of the substances, but this effect is observed with difficulty in liquids, since their molecules change places under changes of temperature. The propagation of heat in these depends on this continuous displacement in all cases where the molecules of the mass are most exposed to the action of the source of heat. If, on the contrary, the source of heat be applied to that part of the mass which is highest, as was the case in several of our experiments, the transfer of heat, which is very slow, does not produce any displacement, at least when the increase of temperature does not diminish the volume, as, indeed, noticed in singular cases bordering on changes of state."

To these quotations from Fourier may properly be added the statement of Maxwell, that "the phrases radiation of heat and radiant heat are not quite scientifically correct and must be used with caution. Heat is certainly communicated from one body to another by a process which we call radiation, which takes place in the region between the two bodies. We have no right, however, to speak of this process of radiation as heat. We have defined heat as it exists in all bodies, and we have seen that all heat is of the same kind. But the radiation between bodies differs from heat (1st), in not making the body hot through which it passes; (2d), in being of many different kinds. Hence, we shall generally speak of radiation, and when we speak of radiant heat we do not mean simply the existence of a new kind of heat."

We close this enunciation of fundamental principles relating to heat with Ganot's statement of *Prevost's Theory of Exchanges:*

"All bodies, whatever their temperature, constantly radiate heat in all directions. If we imagine two bodies of different temperatures placed near each other, the one at a higher temperature will experience loss of heat, its temperature will sink, because the rays it emits are of greater intensity than those it receives; the colder body, on the contrary, will rise in temperature, because it receives rays of greater intensity than those which it emits. Ultimately the temperature of both bodies becomes the same, but the heat is exchanged between them, *only each receives as much as it emits, and the temperature remains constant.*

Fourier shows that if we suppose a solid body formed of a homogeneous substance to be included between two parallel and infinite planes, A and B, separated by the distance e, and if the lower surface A has a constant temperature a, while its upper surface B is maintained at a constant temperature b, less than a, and if both a and b retain their values for an infinite time, then for any temperature v at the distance z from the lower plane, the decrease in temperature between A and the point z will be expressed by the equation

$$v—a=\frac{z}{e}(b—a)$$

and the quantity of flow F of heat which crosses a definite part of a section parallel to the base A during a given time will be expressed by the equation

$$F=K\left(\frac{a—b}{e}\right)$$

in which K is the measure of the specific conductivity of the substance from the base A to the section z. He also shows that if the upper surface B permits the escape into the air of a certain quantity of heat just equal to that which crosses any cross section of the solid in flowing from A to B we shall have:

$$F=h\ (b—\beta)$$

where h is the surface conductivity of the metal. This quantity is just equal to the quantity given by the expression

$$K\left(\frac{a—b}{e}\right)$$

or $$K\left(\frac{a—b}{e}\right)=h\ (b—\beta)$$

from which β, the temperature of the exterior medium at the surface of the solid, can be determined when K and h are known.

*Fourier then extends the investigation to the measure of the movement of heat at a given point of a solid, bounded by six rectangular planes, pairs of which are parallel, and gives the following general equation to determine the thermal condition of any point whose position is determined by the co-ordinates x, y and z:

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* See Translation of Freeman, p. 79.

$$v=A+ax+by+cz$$

in which A represents the surface temperature, and a, b and c represent certain coefficients, which are functions of x, y and z, respectively. He finds that the flow of heat along the different axes is:

$$-k\frac{dv}{dx}\,\omega\,dt \text{ along the axis of } x$$

$$-k\frac{dv}{dy}\,\omega\,dt \text{ along the axis of } y$$

$$-k\frac{dv}{dz}\,\omega\,dt \text{ along the axis of } z$$

in which ω represents the area of a circle of infinitely small thickness and having an infinitely smaller diameter, through which the heat passes from the under side to the upper side during the instant dt; and k represents the internal conductivity which holds for this particular section ω.

We thus have in actual practice three definitions of thermal conductivity, viz:

(1) h=the coefficient of surface conductivity.*
(2) k=the coefficient of the internal conductivity for the elevation corresponding to the height z above the lower plane.
(3) K=the mean coefficient of conductivity for the limit between the base A and the height z.

Two facts are of special importance in this discussion — first, that the flow of heat, or the temperature effect shown in the increase of the length of the bar, is a function of the time of exposure; and, second, that the temperature v can only be computed when we know the surface temperature A and the constants h and k. The complete separation of h from k is practically impossible, and the determination of A presents great difficulties. This difficulty is clearly stated by Sir William Thomson, who, in speaking of the flow of heat from a metal into the surrounding medium, says: "Hence, although the water or the air at the very interface of its contact with the metal, is essentially at the same temperature as the metal, there must be a great difference of temperature in very thin layers of the fluid close to the interface, where there is a large flux of heat through the metal, and the temperature of the fluid as measured by any practical thermometer, or inferred from a knowledge of the average temperature of the whole fluid, or the temperature of entering and leaving currents of fluids, may differ scores of degrees from the true temperature of the solid at the very interface."

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* The term conductivity is here used in the sense of flow of heat, and does not necessarily involve the idea of contact with a hotter or colder metal.

It is generally assumed that the thermal conductivity of iron is 80 times and that of copper, 500 times that of water, and that the thermal conductivity of iron is 3500 times, and of copper 20000 times that of air. It may be doubted whether water is the proper unit of comparison, since its thermal conductivity depends on molecular action which is radically different from that which takes place in metals. In any event, since the various methods of measuring the thermal conductivity depend ultimately, as stated by Tait, "upon observation of temperature of the conducting body at different parts of its mass," and since the element, time of exposure, is exceedingly difficult to manage, even if it is not a real variable, it may well be doubted whether we have at present sufficiently accurate data to secure a practical realization of the analytical solution of the problem given by Fourier. This uncertainty is well illustrated in the divergence of the values of the conductivity of copper determined by Péclet, Clément and Augström. Admitting that the value 1.1 found by the latter physicist to be the true one, the value found by Clément is nearly 200 times, and that of Péclet five times too small.

It is generally assumed, also, that the value of the conductivity varies with the temperature, but we can hardly define the second term of the law in question, as long as so much doubt remains concerning the first term.

The value of the thermal conductivity of different metals, determined by Péclet, Despretz, Tait, Weidemann, and Franz, were founded on the uniform conduction of heat across slabs of metal and along their bases by measuring the absolute quantities of heat that were *conducted out of the substances.* On the other hand the measures of the thermal conductivity, made by Thomson and Forbes, were founded on the laws of the diffusion of heat through bars of iron and of copper placed under ground. Hence, since the conditions of outward flow from the interior are different from the inward flow from the air into the interior, it is evident that the constant of surface conductivity h, must have a different signification and different values, according to the direction of the flow. Calling the outward flow h, we may call the inward flow h^1. Thomson calls the constant h^1 the thermal diffusivity. It expresses the ratio between the thermal capacity and the conductivity of the metal, since the thermal effects are inversely as the thermal capacity of the body.

The same distinction holds in the case of the outward surface flow h, which may be properly called the coefficient of thermal emissivity, or simply the emissivity, according to Thomson.

It is evident that the method of Thomson is more nearly applicable to our present problem, viz., that of the flow of heat in bars of metal having widely different capacities for heat and subjected to varying changes of temperature under air contact. Mathematically considered our problem is this:

Given two or more bars of different specific capacities for heat, supported in air, and having a constant temperature v; what will be their relative thermal conditions after they have been submitted to another constant temperature v^1, for a time T.

The quantity of heat which flows per unit of time across any transverse section of the bar will be the product of its surface conductivity h, of the area of the cross section, and of the gradient of the temperature of that cross section, which may be taken as equal to the difference of the temperatures of the two sides of a layer of unit thickness. If we represent the distance of the first layer from the surface by x, the temperature will be $\frac{dv}{dx}$ and the inward flow will be $\frac{h^1dv}{dx}$. For any distance x+dx the flow becomes

$$*\frac{h^1dv}{dx}+\frac{d}{dx}\left(\frac{h^1dv}{dx}\right)dx.$$

The rate at which the temperature will change will be expressed by $\frac{dv}{dt}$. If we represent by c the thermal capacity of the surface, the flow will be expressed by $\frac{cdv}{dt}$.

Hence $$\frac{cdv}{dt}=\frac{d}{dx}\left(\frac{h^1dv}{dx}\right)$$

For equilibrium we have $\frac{dv}{dt}=0$ and $\frac{d}{dx}\left(\frac{h^1dv}{dx}\right)=0$ and hence we have a linear equation of the form—

$$v=A+Bx$$

in which A represents the surface temperature, and B represents the ratio at which the temperature changes per unit of distance in a direction perpendicular to the surface of the bar.

We here meet with precisely the same difficulty as before, viz., our inability to measure the surface temperature at the very interface and to separate the thermal conductivity B from the constant A.

It is obvious that if these theoretical considerations hold true, we do not obtain the real quantity A, which we desire, by immersing the thermometers in wells sunk in the bars and filled with mercury. This statement is true independently of any question which may be raised with reference to the ability of mercury to take the temperature of the interior mass of the metal.

That the outward and inward flow A, measured by contact of the mercury bulb of the thermometer with the surface of the bar, represents the temperature-effect in changing the length of the bar is shown pretty conclusively by the following experiment. A bar of metal is placed upon the comparator, supported in air at its neutral points and at a constant temperature, *e.g.*, of 32° Fahr. The presence of the observer in the vicinity of the bar will have no apparent effect in changing the length of the bar for a period of fifteen or twenty minutes. But if the observer presses his hand upon the upper surface of the bar for 30 seconds a certain amount of heat will be conducted into the bar *which will produce nearly the entire change in length which will take place, almost at the very instant of contact*, showing that equilibrium between the internal heat imparted and the irradiation at points on the surface different from the point of contact was established nearly instantaneously.

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* See Tait on Heat, p. 219.

If the hand is removed, the shortening due to cooling, will become evident within fifteen or twenty minutes in the case of the bronze bar R_2, and in a time a little less in the case of the steel bar R_3. The heat effect will be entirely dissipated in the course of three or four hours. If the experiment is tried upon a thin strip of metal with large surface area, e.g., with a thickness of one hundredth of an inch, the return to the original condition will occur within a very short time.

If, continuing our experiment, the hand is again pressed upon the bar, after an interval of five minutes, for a second interval of 30 seconds, the bar will become instantly charged with a second increment of heat, which will result in another increase of length, nearly, but not quite, equal to the increment due to the first charge. As many as ten charges have been made at intervals of five minutes, after which it was found that the total change in length was very little different from ten times the amount due to the first charge. It was also quite unexpectedly found that the time required for the dissipation of the ten charges did not differ much from the time required for the dissipation of the first charge.

We are now prepared to return to the inquiries made upon page 17. We have shown that we do not need to determine by what particular process and under what laws internal conduction has taken place in our bars, since the temperature-effect of any increase in temperature in increasing the length of the bar has, to a large extent, its equivalent in the amount of heat which passes the surfaces of the bars by an inward flow, and since an equilibrium between the temperature of the entire mass of the bars with that of the surrounding air has become established. Moreover, under the theory of exchanges, when this equilibrium occurs, *we may neglect the effect of radiant heat altogether.*

We may therefore, employ the mercurial thermometer, placed horizontally upon the surface of the bar, to measure the amount of heat which produces the linear expansion observed. Even if this conclusion should be denied, and it should be insisted that liquid contacts are to be preferred, it may still be shown that it is well nigh impossible to determine with sufficient precision the time when the entire mass of a surrounding liquid is in equilibrium with the entire mass of the metal submerged. This remark applies both to the bars to be compared and to the thermometers employed in making the comparisons when the quantity required is the *real temperature of the liquid.*

This conclusion may be fairly drawn from quotation (1) from Fourier. In actual practice it will be found that while the first effect of liquid contact with a bar of metal is to cause it to approach the normal relation which will exist when equilibrium is established with great rapidity, this relation is reached *permanently only when the entire mass of the liquid and that of the metal have reached the temperature of the surrounding air.* It will be found that the time required for this condition to be fulfilled is considerably greater than that required for the metal to reach the same condition under air contact. When two bars of different thermal capacities are submerged in water at a temperature lower than that of the surrounding air, the error in the determination of their relative lengths will, in spite of every precaution, tend uniformly in one direction. When, under similar conditions, the temperature of the liquid is higher than that of the surrounding air, the actual error of the derived relation will tend uniformly in the opposite direction.

The method of investigation pursued in our first inquiry has been to remove given masses of water and of mercury, placed in vessels having different diameters and composed of different materials, from a nearly constant low temperature to a comparing room in which a nearly constant air temperature could be maintained. The process was then reversed, the liquids passing from a nearly constant air temperature to complete immersion in melting ice. These experiments are still in progress. The details for two exposures are given in exhibit (A). The discussion of the results is properly omitted until the completion of the entire series of observations. This series will include exposures in vessels of glass, tin, ebonite, etc., in vessels both open and closed at the top and covered with blackened caps.

Our second inquiry may be introduced by the relation of an experience of the writer in repeated comparisons of the relative lengths of the steel and bronze standards, made during the years 1884 and 1885. In January of 1884, the writer constructed upon Jessup's steel, three combined yard and meter units of length of the form described in Vol. XVIII., page 290, of the Proceeding of the American Academy of Arts and Sciences. Yard and meter B was made for Professor Brackett, of Princeton College. Yard and meter A_4 was constructed for Professor Rowland, of Johns Hopkins University. Yard and meter R_3 was retained for my personal use. In the discussion of these standards it was found that the equation between the Brackett standard B, and my bronze standard R_2, determined in 1883, differed by a sensible amount from the relation found in 1884. It was also found that the relation between R_2 and R_3, determined in the early part of 1883 differed materially from that found in the latter part of the same year. In order to obtain additional data in regard to the constancy of the relation between these three standards, Professor Rowland kindly returned to me standard A_4 in order that I might be able to verify the comparisons with R_2, made in 1884. The results of the new series are given in exhibit (B). The equations given relate to the relations between standards R_3 and A_4 and the *Metre des Archives* A_0 for the meter, and with the Imperial Yard Y for the yard. With regard to the results of this series I repeat here the conclusions stated in my paper on *The determination of the relative lengths of eight Rowland gratings, printed in the Proceedings of the Society for 1885, page 161.*

"It may be said that the disagreement in the results given in exhibit (B) is not much greater than the probable error of observation, except in the case of the equation $R_3 - 1.2\mu = A_0$. Here the agreement between the separate comparisons forbids the assumption of accidental errors as the cause of the variation. Besides, the disagreement between the different values of the relation $R_2 - B$, omitted here on account of the incompleteness of the first set of observations, is even greater than for $R_2 - R_3$.

In all of the comparisons made since 1884, especial pains have been taken to eliminate the error arising from the failure of the thermometer when placed upon the upper surface of the standards to indicate the real temperature of the metal. Even when a thermometer reading seems to be stationary, it has a *drift* up or down. The direction of this drift is always noted, and the aim has been to combine the observations in such a manner as to balance the errors due to this cause. Very gradually, however, the conviction has been forced upon the writer, that when the temperature of a considerable mass of metal is obtained from the reading of a thermometer placed upon its surface, errors of long period are always introduced which escape detection in a short series of observations, but which manifest themselves in a long series in passing from winter to summer and *vice versa.* This view must at present be taken as a working hypothesis; but if it should prove to be correct, it will have an important bearing upon the work already done in metrology.

In order to test this theory it is absolutely necessary that the comparisons shall be made in a room in which a steady temperature can be maintained for a long period of time, and in which the changes which occur are very slow. Otherwise, it would be impossible to separate the errors of short period from those of long period. Through an appropriation from the Rumford Committee of the American Academy of Arts and Sciences, a comparing room was built beneath the rotunda of the Observatory which nearly fulfulls the required conditions. The full account of the work undertaken by Mr. A. L. McRae and by the writer, under this appropriation, will appear in a report to the Academy, but by permission the results of the comparison between R_2 and R_3 are given here.

Arranging the comparisons in groups of ten each, we have the following result:

From May 25 to May 27, $R_2 - R_3 = +5.32u$
" May 28 to May 31, $= +4.16$
" May 31 to June 2, $= +4.81$
" June 3 to June 4, $= +4.00$
" June 5 to June 16, $= +4.05$
" June 16 to June 27, $= +2.23$
" June 28 to July 8, $= +2.34$

If these observations can be trusted, we have here a decided diminution in the value of the relation R_2 R_3 between May and July, notwithstanding the most careful efforts to eliminate all sources of error. The temperature during this interval gradually increased from 9° to 15° C. It is also to be distinctly noted that these variations *inter se* are much greater than we should naturally expect under the variations of temperature which ordinarily occur.

It is evident that we have here under another form, a problem which has, so far, baffled every attempt at solution. The difficulty encountered is well illustrated in the diversity of the results which have been obtained for the relation between the yard designated "Bronze 11," which is the virtual standard of the United States, although it has received no legal sanction, and the Imperial Yard of Great Britain. When this yard was presented to the United States by the British Board of Trade, in 1856, it was declared to be standard at 61°.79. It was, therefore, fourteen hundred thousandths of an inch *too long*. Within a few years two series of direct comparisons between these two standards have been made as the result of which it appears that "Bronze 11" is eighty-eight millionths of an inch *shorter* than the Imperial Yard. The discrepancy has been regarded as a well-authenticated instance of an actual change in length; but, Pierce, in 1882, found "Bronze 11" to be only eleven millionths of an inch shorter than the Imperial Yard. The evidence of change of length due to age of the metal is, therefore, not conclusive, unless it is admitted that changes in two directions have taken place.

It became evident, early in this investigation, that while a given series of comparisons might give results which would agree *inter se* within very narrow limits, another series made at a different time of the year, would give a final result, which, in many cases, differed from the first result by an amount far greater than that due to the accidental errors of observation. It soon became apparent that these observed variations were confined to bars of metal having widely different coefficients of expansion, and having widely different masses.

During the latter part of the year 1885, the necessity for an organized search for the cause of these abnormal variations became so manifest that it was deemed useless to continue further comparisons of the standards of length under the system employed up to this time. There were many evidences that the abnormal variations in question were in some way connected with the indications of the temperature by the thermometer employed. It was therefore resolved to make a special study of the individual thermometers, simply as instruments designed to record the temperature and without any relation to their use in connection with comparisons for length.

The investigation which follows was made possible by the construction of a comparing room beneath the rotunda of the Observatory of Harvard College, from the avails of an appropriation made for this purpose by the Rumford Committee of the American Academy of Arts and Sciences. The details of the work will appear in a report to the Academy, but by permission, the general results of the investigation are given here.

The following working hypothesis was assumed as a basis for this investigation. The height of a column of mercury in the tube of a thermometer depends on the quick action of the temperature in expanding the mercury in the bulb, and the slow action upon the column of mercury, which takes place through changes in the temperature of the stem, and especially of the metal framework upon which the thermometer may be supported. The resultant of these forces, acting sometimes in unison with each other, but more often in opposition to each other, produces slight oscillations in the height of the column, which vary with the relative masses of the bulb, including the mercury and the stem. A thermometer whose bulb is composed of a very thin shell may be very sensitive, but this good quality may be neutralized by the liability to changes due to the heat or cold stored up in a thick stem. Pulsations may also occur on account of the abrupt variations in the diameter of the bore. The action here indicated is to be distinguished from the instantaneous fall of the mercury column due to a very sudden increase of temperature resulting in an increase of the diameter of the bulb before the change has time to act upon the mercury within the bulb. In any event, any particular thermometer selected is liable to contain idiosyncrasies which must be carefully studied before reliance can be placed upon its indications.

In order that the conclusions reached might, if possible, be applicable to thermometers as a class rather than to individual thermometers, the following thermometers were placed in the series to be compared.

(1) A Yale Centigrade standard, by Tonnelot, designated Y_{62}.

(2) A Normal Centigrade standard, by Warmbrum, Zuilitz & Co., of Berlin. This thermometer, designated G_1, is the property of Wellesley College.

(3) A Normal Centigrade standard, by Weigand, of Wurzburg. It is designated G_2 and belongs to Professor Goodale, of Harvard College.

(4) A Normal Fahrenheit, by Baudin, designated B and standardized at Breteuil.

(5) A spirit thermometer, by Huddleston, of Boston, designated H.

(6) A spiral bulb thermometer, also by Huddleston, designated S.

(7) A Fahrenheit standard, by Green, designated G_2.

(8) A Fahrenheit standard, by Cassella, of London, designated C.

(9) Three spiral metal thermometers on the Briguet principle, made by the Standard Thermometer Co., of Peabody, Mass.

Since it is at least a plausible supposition that there may be errors peculiar to all mercurial thermometers as a class, a form of metal thermometer was employed from which fairly good results were obtained. Strips of steel and silver were stretched from a common point of support side by side, the tension being produced by spiral springs. The strips were one inch in width and their thickness was not far from one hundredth of an inch. At the distance of one meter from the point of support a common line was traced across the surface of each strip. It was therefore possible to measure the relative expansions of the steel and the silver strips by measuring with the micrometer of the microscope, the amount of separation between the two lines.

This form of thermometer was found to be far more sensitive to sudden changes of temperature than the mercurial thermometers, but the certainty of the return to the zero point after wide excursions, cannot be considered as definitely established. The degree of constancy will be seen from the record given in exhibit (C).

Inasmuch as the strip thermometer was found to be much more sensitive to changes of temperature than the mercurial thermometers, the agreement of the separate values of Δ is quite as close as ought to be expected. It will be seen that the values of Δ are larger than the mean value when the temperature is rising, and smaller than the mean value when it is falling. From ten observations with a rising temperature the value of Δ is 64.7 divisions of the micrometer. From ten observations with a falling temperature $\Delta = 57.2$ divisions. The mean value is 59.9 divisions, which agrees closely with the mean value derived from the observations made at a stationary temperature, viz., 57.7 divisions. In this series therefore the zero seems to be constant. It should be said, however, that in several preceding series the departure from the established zero occasionally amounted to nearly 1°, especially when the tension of the strips was evidently too great. It will be seen from the solution of the equations of condition that the final residuals indicate a liability of error which does not much exceed two tenths of a degree.

The entire number of comparisons of thermometers made during the year 1886 is about 22,000. Since this work is to a large extent in the nature of a diligent search after unknown quantities, the observations have been divided into a series of groups, in each of which some particular inquiry has been the leading feature. The conclusions reached in each series will be given in the order of the observations upon which they are based. It will be observed that these conclusions are, in several cases, modified by subsequent observations and it is quite likely that further modifications will be necessary.

Our problem may be stated as follows:

Can the variations of standards of length from their normal relations, observed at different seasons of the year, be attributed to systematic errors in the indications of the thermometers with which the temperature is determined?

The normal relations between the different thermometers, compared under normal conditions, were first obtained in order that they might serve as a basis of comparison for the abnormal relations. The results obtained depend upon about 8000 comparisons. An examination of these results shows that we must always expect errors of reading varying from 0.1° to 0.3° C, but these errors are largely due, as shown by the observations of 1887, to the lack of attention to the necessity of making the comparisons at the time when there is an equilibrium between the temperature of the thermometers and that of the surrounding air.

There can be no better test of the character of the indications given by a thermometer, than the determination of the time required to pass from a normal reading at one constant temperature to a normal reading at a different constant temperature. The determination of this time, for the different thermometers, constitutes our first inquiry. The method of observation was as follows: All the thermometers exposed were allowed to come to rest under one of the conditions selected, which we will call condition A. One of the standard thermometers, generally Y_{62} remained permanently at the temperature of the condition to which the thermometers were to be removed, which is designated condition B. After the removal of all the thermometers, except one from A to B, the comparisons with the thermometer at B were made at intervals of about five minutes. After a complete equilibrium between all the thermometers had been established for condition B, they were restored to condition A, and similar comparisons were made with the standard which remained under condition A.

SERIES I — Condition A. Thermometers suspended from wires in open air at — 10° Fahr.

Condition B. Thermometers rest horizontally upon a shelf in the Prime Vertical Room of Harvard College Observatory, in which a very constant temperature of about 5° Fahr. was maintained for several days.

Conclusions. Normal relations reached in times varying between nineteen minutes and four hours. Movement by pulsations at several points very marked. The column of mercury often stationary for a long time, followed by sudden upward or downward movements. The mercurial thermometers all pass the point of their normal relation in times, somewhat proportioned to the mass of the mercury in the bulb, and the thickness of the stem. These times vary between 19^{m} and 25^{m}. The time required to overcome this excess of reading varies from 1^{h} to 4^{h}. All the thermometers have nearly the same degree of sensitiveness, except the Cassella standard. The metal thermometers reach the normal relation a little in advance of the mercurial. The metal and the mercurial thermometers agree closely at the low temperatures.

SERIES II — Condition A, same as condition B in Series I. Temperature — 10° C.

Condition B. Thermometers rest horizontally upon a shelf in the clock-room of the Observatory, in which the variations in temperature during twenty-four hours does not much exceed 0.5°.

Conclusions. All the thermometers, both mercurial and metal, pass the normal point, and the time occupied till the return varies between two hours and forty-eight hours. In two or three instances the normal point was passed a second time in the opposite direction. Slow oscillations occur at different points and with great irregularity in regard to time. Y_{62} always reaches the normal point in advance of the other thermometers; the spirit thermometer, which has a very large cylindrical bulb, always falling behind, but the difference does not much exceed four or five minutes. The average time required to recover the distance passed beyond the normal point is nearly the same in all cases, except with the Cassella thermometer. *First noticed that the amount of running by, and the length of time required for the return is, to a certain extent, proportioned to the range of temperature passed over, but is proportionally greater for slow than for rapid changes.* A large number of instances occur, in which all the thermometers agree in giving erroneous results after passing the normal point varying in amount from 0.2° to 0.4° C. The persistence with which these abnormal variations occur shows that they are not due to accidental errors of observation.

SERIES III — Condition A. Thermometers immersed in melting ice.

Condition B. Thermometers rest horizontally on metal bars, which rest upon the comparator in the new comparing room beneath the rotunda of the Observatory. Mean temperature about 15° C.

Conclusions. Time of passing from 0° to 15° varies from 17 to 25 minutes. Marked evidence of the truth of the inference, drawn from Series II, that the abnormal variations which occur after the thermometers pass the normal point are much greater for very slow variations than for rapid changes, and also that the time of recovery after passing the normal point is much greater for a change of 15° C. than e. g. for a change of 30° C.

SERIES IV — Condition A. Thermometers immersed in melting ice.

Condition B. Same as in Series III, but the constant temperature of the comparing room was about 38° C.

Conclusions. Very slight evidence of passing the normal point. The time required to pass from 0° to 38° nearly the same for all the thermometers except Cassella and the Huddleston spiral thermometer, viz., from 17 to 25 minutes. Metal thermometers less sensitive than the mercurial for a few minutes after removal, but they reach the normal point in about the same length of time. This series offers pretty conclusive evidence that the time required to pass from one temperature to another constant temperature is nearly the same whatever the range of temperature passed through, but that the recovery after passing the normal point varies inversely as the range and directly as the relative masses of the bulb and of the stem.

SERIES V. The thermometers in this series were supported in air at their neutral points upon pieces of tin set edgewise.

Conclusion. The most important result from this series is the apparent demonstration of the fact that under a gradually increasing temperature all the mercurial thermometers read too low by amounts varying between 0.1° and 0.4° C. and too high by about the same amounts during a slowly decreasing temperature. In other words they fail to take the temperature of the surrounding air while these slow changes are occurring.

SERIES VI — In this series part of the thermometers rested horizontally upon bars of metal and part were supported in air.

Conclusions. Under slow changes of temperature there is a very decided evidence that the temperature of the bars hold the thermometers which rest upon them in check, and that they only reach the normal relation after several hours. The strip thermometers give indications far in advance of the mercurial thermometers, even indicating a falling temperature while the mercurial thermometers indicate a rising temperature, and for a period of 20 minutes after the change in temperature takes place.

SERIES VII. This series was undertaken to determine how far, under a rapid rising temperature, the mass of metal upon which the thermometers rest may hold them in check as shown by comparison with a thermometer supported in air at about the same height. On July 4, at $7^h\ 40^m$ A. M. it was found that thermometers G_1 and G_2 read 0°.93 and 0°.73 respectively higher than the corrected value of Y_{62}. These values represent the real corrections required at 15° C. G_1 was supported in air, while G_2 and Y_{62} rested upon a large mass of metal, weighing about 100 lbs. At $8^h\ 11^m$ the gas under the boiler outside of the comparing room was lighted. The hot water from this boiler had a steady flow through iron pipes in the comparing room, returning to the upper part of the tank. The effect of the mass of iron in holding thermometers Y_{62} and G_2 in check will be seen from exhibit (D).

It appears from this experiment, that, under a rapid change of temperature in the comparing room, thermometer G_1 in air was nearly 8° in advance of Y_{62}, while G_2 was only a little over 1° in advance after an interval of 2 hours. This excess continued with a diminishing ratio for nearly an hour after the heat had been withdrawn from the boiler. At twelve o'clock, four hours after the commencement of the series, G_1 was 3°.5 ahead of Y_{62} and 0°.6 ahead of G_2. At no time, therefore, during an interval of 4 hours, did either of the thermometers, placed upon the iron, indicate the temperature of the surrounding air.

While the observations made during the year 1886 were of great service in indicating the conditions under which temperature should be determined, it cannot be said that they offered any decisive evidence in regard to the real solution of the problem in hand.

It was, therefore, determined to make a new series of comparisons under entirely different conditions. The series for 1887 was made after my removal from Cambridge to Waterville, Me. The plan of observations involved the comparison of 30 Fahrenheit thermometers of the Signal Service pattern made by Green, of New York, with centigrade standard Y_{62}, and a Green centigrade standard.

These thermometers were divided into six groups of five each. In the in-door series they were suspended at intervals of about two inches between the groups, from a long bar which was itself suspended near the middle of a large room, at the distance of about five feet from the floor. In the out-door series the thermometers were suspended upon a revolving frame about ten feet from the ground. The standard cylindrical bulb thermometers were suspended horizontally, first a little below the spherical bulbs of the Fahrenheit thermometers, and afterwards at different distances above the bulbs, The readings were at first made with an objective of two inches aperture from a distance of about twelve feet, and afterwards with a telescope of four inches aperture and from a distance of forty feet. Thus far, about 17000 comparisons have been made, and it is proposed to continue the observations for one year. Already comparisons have been made between the extreme temperatures of $-30°$ and $+95°$ Fahr.

In this series of observations a radical departure from former methods of comparisons was made.

Reasoning from the laws of the flow of heat between the thermometers and the surrounding air enunciated by Fourier, it seemed to me that most, if not all the errors due to our imperfect knowledge of the laws of the conduction of heat, and especially to our inability to distinguish between the flow of heat from the interior of a mass of metal outward from its surfaces into the air, and the radiant heat which escapes from the surface during slow changes of temperature, would be at least reduced to a minimum, if all the comparrisons were made at the critical point of equilibrium, or, as I may call it, the critical point of quiescence when the entire mass of the thermometer is at the same temperature as that of the surrounding air.

Under the third inquiry it appears, from the observations given under exhibit (E), that when steel and bronze bars are compared while the air is undergoing a slowly rising temperature, the steel standard, which has a less cross-section than the bronze, becomes relatively *longer* than the bronze, while for a slowly decreasing temperature it becomes relatively *shorter* than the bronze standard. The difference may amount to as much as five or six mikrons. Reasoning in the same way, as in regard to the thermometers, it seemed reasonable that at the point of perfect equilibrium between the temperature of the air, the entire mass of the metals and of the thermometers placed upon their upper surface, we ought to obtain nearly an ideal condition in which the effects of conduction, radiation and irradiation need not be considered. At any rate, it seemed possible to eliminate the errors in question by combining observations made, as nearly as possible, at equal distances on each side of the critical point of no variation. By determining this critical point for air and for various liquids, we ought, in every case, to obtain the same result as would be obtained if we could employ an ideal fluid and a thermometer of ideal properties under conditions in which actual changes of temperature would be recorded instantaneously. Now two of these critical points of equilibrium occur in nearly every day in the year: one in the early morning, and the other late in the afternoon. It was not found possible to make use of the one which occurs in the afternoon in our particular problem; but in the early morning this point is well defined. At Waterville, under the conditions which there prevailed, at least during the years 1886–7, the time of equilibrium occurs about half an hour after sunrise, on clear days, and somewhat later on cloudy days. The time for thermometers is considerably earlier than for the standard bars.

Nearly all of our difficulties therefore ought to vanish by the adoption of the simple plan of making all comparisons, both of thermometers, and of standards of length symmetrically with respect to the critical point of quiescence.

After an experience of several months I feel safe in saying that they *do* vanish.

That part of the discussion which relates to the comparisons of the thermometers themselves *inter se* is now no longer germain to our present inquiry. It will be reserved therefore for a subsequent communication. It will be sufficient to state here four of the conclusions which seem to be established by the observations which have been made up to the present time.

(a) The time required for thermometers to pass from one constant temperature to another temperature also constant, varies to a certain extent with the form of construction of the thermometers, and is nearly the same whatever the range of temperature. For thermometers Y_{62}, B and G_1 this time is not far from 17 minutes. The longest time required is about 26 minutes. The time required for the spiral metal thermometers is about 20 minutes, while for the metal strip thermometer it is decidedly less than for the mercurial thermometers.

(b) The tendency to pass the normal relation during a change from one constant temperature to another constant temperature, is due to a far greater extent than was at first suspected to the conduction effect of contact of the thermometers with metals of large mass, upon which the thermometers rest. The supposition that the slow action of heat through the stem and especially through the metal framework of the thermometers would account for the slow oscillations of the mercury column observed, still holds true, but to a less extent than at first supposed, the larger share of this effect being due to the control which the temperature of the metal bars upon which the thermometers are placed holds over the temperature registered by the thermometers.

The truth of this statement is clearly shown by noting the time in which a series of thermometers will pass from 0° C. to the constant surface temperature of a large mass of metal for the angular positions of the stem with respect to the metal at 0°, 20°, 40°, 60° and 90°.

(c) Admitting that the real temperature at any instant, is registered by the cylindrical bulb thermometers Y_{62} and G, the first indications given by the spherical bulb thermometers with a supporting metal frame-work are entirely untrustworthy. Careful observations show that when a change from the stationary point of equilibrium takes place, as indicated by Y_{62} and G, the readings of 30 Fahrenheit thermometers pass from a variation, not exceeding 0.1°, to a variation of about 0.8°. The extreme normal mean deviation of these 30 thermometers at 30°, 50°, 60° and 80° from Y_{62} and G, is certainly less than 0.1°, and yet within 30 seconds after a change has been noted in Y_{62} or G, they often exhibit deviations of from two or three tenths of a degree between different groups, and of from six to eight tenths of a degree between the individual thermometers. In case the temperature again becomes constant and remains so for about fifteen minutes the original agreement is restored.

The same result has been obtained by having an assistant walk back and forth behind the thermometers in equal intervals of time, while the effect of the heat thus communicated was observed with the aid of the reading telescope.

(d) The indications given by thermometers which are themselves undergoing changes of temperature, are uncertain within limits varying between 0.1° and 0.4° C. The necessity of making all comparisons at the stationary point of no variation of temperature is quite as great as for the comparison of standards of length under the same conditions. The length of time, however, during which the temperature must remain constant before the comparisons can be safely made, appears to be about 20 minutes for air contacts. The time required for liquid contacts depends on whether the object is to compare thermometers *inter se* under identical conditions, or, which is quite another thing, to determine the real temperature of the liquids in which the immersion takes place. The experiments, which are now in progress, point unmistakably to the conclusion that equal quantities of water and mercury pass from a given constant temperature to another temperature, also constant, in nearly the same time, but with widely different rates of flow. If subsequent observations confirm this conclusion, it is evident that we may substitute for the specific heat of a substance which Thompson defines as the thermal capacity of a stated quantity of it, the specific rate of flow of heat for any substance.

The application of this method of making a mechanical balance of the heat effects of varying temperatures has resulted in the apparent establishment of the real constancy of the relative coefficients of expansion between Bailey's metal, Jessup's steel, and Chance & Son's glass between the limits of —5° and +95° Fahrenheit. 293 sets of comparisons have been made, extending from Dec. 18, 1886, to July 14, 1887. About one-half of this number were made from one-quarter to three-quarters of an hour after sunrise on clear days. Perhaps one-eighth of the entire number were made on days when there was a very steady temperature throughout the entire day, and it is to be especially noted that the mean of the entire series of balanced values is nearly the same as the mean of the values derived from the observations on rainy days.

The limitation of the time of closing the series was made with a definite object in view. Until all but the last two series of observations had been completed, viz: from —5° to +65°, I had no knowledge of the results to be expected. In May of the present year, the observations made up to this point were reduced, and I then knew, for the first time, that the laws of constancy was likely to be established. *As the remaining observations went forward I found it impossible to resist the tendency to obtain a result which I expected.* This experience presents a curious psychological question. When any mean value has been really established, there must be after that as many positive as negative corrections, if the observations are continued for a sufficient length of time, and one can obtain, within narrow limits, almost any value desired by *stopping the series of observations at the right time.* In order to obtain, as far as possible, freedom from errors of this class, it was decided, early in June, to close the series on July 14, 1887, and this programme was strictly carried out.

The plan of the work adopted is an exceedingly simple one. It consists:

(a) In obtaining a sufficient number of observations in groups at intervals of about 10° Fahrenheit, in order to neutralize the accidental errors of observations in that group.

(b) In either limiting the comparisons to the times when equilibrium appeared to be established between the temperature of the bars of metal, of the thermometers and of the surrounding air, or in distributing the observations symmetrically about the point of no variation of temperature as determined by thermometers placed upon the upper surface of the bars of metal and also supported in the air in nearly the same horizontal plane.

(c) In making continuous comparisons when the atmospheric conditions were such that a steady temperature could be maintained for several hours at one time as on rainy days. It has been found that the comparisons could be safely continued for an entire day, at intervals of one hour, when the entire rise or fall of temperature did not exceed 0.8° Fahr. Since the general tendency of the temperature would, on such occasions be upward, every opportunity possible was chosen in which the slow drift of the temperature was in the opposite direction. Cloudy days following a change of wind to the north during the previous night, were utilized as far as possible, but it was occasionally found necessary, in order to maintain an equality between the number of upward and downward drifts of temperature, to resort to heating the comparing room by convection for several days, and then to allow a very gradual dissipation of the heat thus stored up.

During this series of observations the comparator was mounted in the third story of one of the College buildings. Contrary to expectation, this room was found to be far better adapted to the control of the temperature than the room beneath the rotunda of the Observatory at Cambridge. The comparator is mounted in a direct line between two large windows, so that it is possible to obtain, practically, open air exposure. The dimensions of the room are 20x12x12 feet. On account of the great thickness of the brick walls and the thoroughness with which the outside air can be excluded by means of heavy shutters which slide into openings in the centre of the walls it is easy to control the range of temperature within narrow limits for any selected time.

While at Cambridge the whole variation, within a given time, was less than at Waterville, it was far more difficult to ascertain, at the former place, the precise point of no variation of temperature, and hence there was far greater danger that the systematic errors in question would escape detection.

The stop method was employed in all of the observations. The stops having been firmly clamped at a distance apart of approximately one yard, the microscope carriage was first brought into contact with the left-hand stop, and two readings of the microscope for coincidence between the micrometer line and the initial line of the standard were taken. The carriage was then brought into contact with the right-hand stop, and four readings for coincidence were taken. As a check upon the constancy of the parts of the apparatus involved in the entire forward and backward movement of the carriage which carries the first microscope, two readings were again taken for the first contact. It is evident that the difference between the readings will give the length of the bar with respect to the distance between the stops, and also that when the difference between this value and the corresponding value for another standard is taken, the distance between the stops will be eliminated.

The following example will serve as an illustration:

R_2 with stops.

Left.	Right.	R—L.
7 rev. 42.1 div.	7 rev. 40.8 div.	—1.3 div.

R_3 with stops.

Left^1.	Right^1.	R^1—L^1.
7 rev. 37.8 div.	7 rev. 35.0 div.	—2.8 div.

When R—L or R^1—L^1 has the minus sign the interpretation is that the length of the standard is greater than the distance between the stops. In this example yard R_2 is 1.3 divisions and yard R_3 is 2.8 divisions longer than the distance between the stops.

If we subtract R—L for R_2 from R^1—L^1 for R_3 we have:

$$(R^1-L^1)-(R-L)=R_2-R_3.$$

In this case $R_3-1.5\text{ div.}=R_2$

indicating that R_3 is 1.5 divisions longer than R_2.

The results of the comparisons between R_2, R_3 and G are given in exhibit (E). The glass bar G has five separate yards laid off upon its surface, each being the result of corrections derived from the one previously ruled. Yard G_1 has lines which have a width of about 5 mikrons, while the width of the lines of G_5 is but little over 1 mikron. Both G_1 and G_5 were used in all the comparisons in order to ascertain whether any personal equation exists which depends on the size of the lines.

The results of the comparisons are expressed in divisions of the micrometer in which

$$1\text{ division}=0.567\mu.$$

In the discussion of the observations the mean result for each group of ten sets is compared with the result from the next succeeding group, as shown in exhibit F. The separate results for the value of the relative coefficients, expressed in mikrons, are as follows:

Limits of temperature.		Relative Coefficients.		
		$(R_2—R_3)$	$(R_2—G_1)$	$(R_2—G_5)$
4°.48	15°.53	3.53μ	4.70μ	4.77μ
15.53	26.05	3.52	4.64	4.54
26.05	35.54	3.48	4.50	4.62
35.54	45.18	3.52	4.31	4.28
45.18	54.52	3.58	4.65	4.65
54.52	64.94	3.55	4.77	4.78
64.94	75.21	3.53	4.69	4.65
75.21	85.09	3.38	4.48	4.40
	Means.	3.51μ	4.59μ	4.59μ

Three facts will at once arrest the attention upon an examination of the results in this table.

(1) There is no well defined evidence of an increase in the value of the relative coefficients, corresponding to an increase in the temperature.

(2) There is an exact agreement between the values of the coefficients derived from G_1 and from G_5.

(3) The probable errors of observation are much greater for the glass bar than for the steel bar. This is due to the greater mass of the glass bar and to the consequent lack of the precision with which the time of equilibrium can be determined.

In exhibit (G) the comparisons are arranged in groups of 10 observations each. In the first half of the series the residuals $R_2—R_3$ and $R_2—G$ are reduced to the mean temperature 33°.86 with the relative coefficient first obtained, viz: 6.17 div. for $R_2—R_3$ and 8.09 div. for $R_2—G$. In the second half the reduction is made to the mean temperature 73°.84. The solution by Least Squares of the equations of condition found from these observations, gives the following values for the absolute coefficients:

	R_3	G_1	G_2
First half of series	5.213μ	4.189μ	4.179μ
Second half of series	5.278	4.153	4.178

While the result for R_3 shows a slight increase for an increase in the value of the temperature, the glass bar indicates a corresponding decrease. The agreement here indicated is greater than we ought to expect except when the number of comparisons is very large.

In exhibit (H) all of the observations are reduced to $62^{\circ}.0$. The solution of the equations gives:

For R_3 coefficient $= 5.205\mu$
" G_1 " " $= 4.126\mu$
" G_5 " " $= 4.127\mu$

When these values are reduced to the corresponding values for the centigrade scale we have the following results, which may be compared with the results published before this series was commenced in the Proceedings of this Society for 1885, page 157, and in the Proceedings of the American Academy of Arts and Sciences for 1882–3, page 385.

	From the present series.	From previous observations.
Coeff. of R_3	$= 10.250\mu$	$= 10.246\mu$
Coeff. of $\frac{G_1+G_5}{2}$	$= 7.428$	$= 7.400$

It appears also from this solution that yard R_3 is 9.78 divisions or 5.55μ longer than R_2 and that G_1 is 1.07 divisions or 0.61μ longer than R_2 at $62^{\circ}.0$ Fahr. In order to compare these values with those previously obtained it will be necessary to apply the corrections due to the horizontal curvature of the ways of the comparator. It will be remembered that the standards were arranged side by side upon the comparator. The distance from R_2 to R_3 was 19.7 mm. and from R_2 to G was 53.4 mm. The correction for the horizontal curvature is 5.40 divisions for each centimeter of distance from the central line between the ways

We have therefore:

$R_2 - R_3 = -9.8$ div. $+ 10.6$ div. $= + 0.8$ div. $= + 0.5\mu$.
$R_2 - G_1 = -1.1$ div. $+ 28.8$ div. $= + 27.7$ div. $= + 16.7\mu$.

By the reference already given it will be found that the corresponding values previously found are:

$$R_2—R_3 = — 0.5\mu$$

$$R_2—G_1 = +17.0\mu$$

It is therefore evident that no change greater than the accidental error of observation has taken place in the lengths of these standards since they were first made.

We are now prepared to make a practical application of the results of our inquiry. Since we can assume the relative coefficients of expansion between the bronze and the steel bars to be 6.20 divisions of the micrometer for each and for every degree of the Fahrenheit scale and of the bronze and glass bars to be 8.11 divisions, we can ascertain the corrections of the thermometers employed to measure the very low temperatures on Jan. 4, Jan. 9 and Jan. 19, 1887. The following observed relations between the lengths of R_2, R_3, G_1 and G_5 are placed beside the values compiled from the data given in exhibit E.

$R_2—R_3$

Date.	Thermometer.	Observed.	computed.	c—o
Jan. 4.	—19°.80	—490.0 div.	—517.0	—27.0
9.	—13.90	—457.0	—480.8	—23.8
19.	—11.85	—459.6	—467.7	— 8.1

$R_2—G_1$

Date.	Thermometer.	Observed.	computed.	c—o
Jan. 4.	—19°.80	—645.8 div.	—683.1	—37.5
9.	—13.90	—614.0	—635.2	—21.2
19.	—11.85	—614.0	—618.6	—

$R_2—G_5$

Date.	Thermometer.	Observed.	computed.	c—o
Jan. 4.	—19°.80	—623.1 div.	—658.8	—35.7
9.	—13.90	—596.3	—616.6	—20.3
19.	—11.85	—588.9	—600.0	—11.1

If we assume the computed values given above to be real, they may be reduced to the observed values by dimnishing the residuals c—o by the quotient of c—o divided by 6.20 for the steel bar, and by the the quotient of c—o divided by 8.11 for the glass bar. We have therefore:

	From R_2—R_3	From R_2—G_1	From R_2—G_5	Means.
Correction at —19°.8 =	+4°.36	+4°.60	+4°.40	+4°.34
Correction at —13.9 =	+3.84	+2.61	+2.50	+2.98
Correction at —11.85 =	+1.31	+0.56	+1.37	+1.08

These results confirm the conclusion reached by an entirely different method of investigation, undertaken by the writer, viz: *That mercurial thermometers, as a class, read too low when the temperature is below minus ten degrees Fahrenheit.*

It must be noted, however, that these observations were made with great difficulty, and definite conclusions can hardly be drawn from them.

ADDENDUM.

I.

The values R_2—R_3 = —9.8 div. and R_2—G_1 = —1.1 div. were obtained from the solution of the equations of condition about ten days before this address was delivered. They were, therefore, unknown to me when the observations were made. Inasmuch as the comparison of any subsequent determinations with these values would offer additional evidence in regard to the correctness of the theoretical considerations offered in this paper, it was thought worth while to make an additional series of comparisons after my return to Waterville after the close of the annual meeting. The entire data for this series are recorded in Exhibit (K). The original readings for R_2—R_3, R_2—G_1, are given because they furnish the means of comparison with the nearly constant value of the distance between the stops of the comparator. The corrected readings of the thermometers are given in columns 2, 3, 4 and 5. No. 1 rested upon R_3, and No. 2 upon G. No. 3 rested upon the edge of a paper box, at about the same height as G, while No. 4 was similarly supported at the height of R_2. In the observations of October 4th, 5th and 6th, a Green standard was inserted in a groove cut in a bronze bar having the same cross-section as R_2. The depth of the groove is half an inch, and the bulb of the thermometer was immersed in mercury. The opening was covered by a thin sheet of rubber placed upon the surface upon which another bar was placed.

All the observations are reduced 62°.0 by means of the relative coefficients 6.2 div. and 8.11 div. The last two columns contain the residuals $[R_2 - R_3] + 9.8$ div. and $[R_2 - G_1] + 1.1$ div.

An asterisk is placed opposite the comparisons which are too far removed from the point of no variation of temperature to be safely included in the determination of the relations between R_2 and R_3, and between R_2 and G_1.

The results of the observation which have been made since September and which have been arranged with reference to a more complete test of the laws relating to the action of heat upon the metals under investigation, enunciated in this paper, than was possible in the earlier comparisons, in which the only object aimed at was an equal and symmetrical distribution with respect to upward and downward *drifts* of temperature, will be recorded in the form of conclusions with the references to the data by which they are established.

It may be well to restate, at this point, the grounds upon which preference is given to air-contact observations, in the light of more recent experience.

(a) In order that the expansion and contraction of metals under variations of temperature may take place under the action of forces for which simple analytical expressions may be obtained, it is essential that the medium by which the heat is communicated to them shall be symmetrically distributed at all points of their surfaces. This condition is indeed fulfilled to a certain extent in liquid contacts, but by no means as perfectly as in air contacts.

(b) The temperature can be kept under far better control in air contacts than in liquid contacts.

(c) As will be presently shown, the air seems to have very little power to communicate to the metals under comparison, the generally higher temperature of the observer. Without doubt the temperature of the air immediately surrounding the observer is raised, but it is not certain that the currents created reach the metals directly. The observations of and Mallet in the ascent of the "Horla" tend to show that currents of air do not readily mingle when they meet, and that colder currents pass under warmer ones instead of mingling with them. Mass, however, is the chief factor of resistance to change in length.

(d) The greater part of the conductive effect in liquid contacts is indeed nearly instantaneous in its action, and metals submerged in liquids quickly acquire the temperature of the immediate layers by which they are surrounded, but the effect of changes in temperature upon the liquid is far more irregular and requires a much longer time for complete saturation than in the case of metals when submitted to the same conditions of temperature. Metals probably change their temperature under the action of a single force. Liquids pass from one temperature to another under the action of at least two forces, viz.: by direct transfer as in the case of metals and under the action of currents. My own observations seem to show that while these currents are at first more marked in the case of water than in the case of mercury the difference is hardly appreciable at the expiration of two hours after the removal from a given constant temperature to a different temperature also constant, but the time required for each to reach complete saturation is nearly the same, viz., from eight to twelve hours. This limit of time is approximately independent of the quantity of the liquids used in the experiment. The ordinary method of maintaining liquids at an apparently constant temperature either by a gentle flow or by vigorous stirring, creates an additional source of error by adding a third force to the two already mentioned, viz., an increase of temperature by friction.

(e) Since a liquid must under ordinary circumstances have a part of its surface exposed to the air, the condition named under (a) must always be violated in the passage of the liquid from one temperature to another.

The following are the conclusions drawn from the observations given in detail under exhibits K, and L.

The limit of error may be safely placed at about 1.0μ. It is doubtful whether two equally careful observers will uniformly obtain results which will show an agreement much closer than this limit; for example, in the comparison of two meters. The errors due to personal equation, and especially to personal equation in focus, are of themselves quite sufficient to demand at least the limit of 1.0μ, even if the graduated surface remains in an invariable plane when the bar of metal is supported at two fixed points. The limit of the relative error in repeated comparisons under the same conditions, may be placed at about 0.2μ.

(1) *In the comparisons of standards of length which have considerable mass, the presence of the person of the observer will produec no sensible effect in increasing their length within the limits of the accidental errors of observation, during the time ordinarily required for a complete comparison.*

The limit of safety may be extended to two hours of continuous work when the air is comparatively free from moisture. If the time required for a single comparison does not exceed five or six minutes the observations may be safely repeated at intervals of one hour without limit in regard to time. An examination of the comparisons between R_2 and R_3, for October 27, exhibit K, will show that under a nearly constant temperature, the normal relation between these standards was maintained between 6^h 10^m and 8^h 25^m. Until 7^h 20^m the interval between the comparisons was about eight minutes. Those made between the limits, 7^h 18^m and 10^h 45^m on September 29, between 6^h 20^m and 9^h 0^m on October 3, between 1^h 45^m P. M. and 3^h 45^m on October 5, between 6^h 30^m A. M. and 4^h 0^m P. M. on October 10, and between 3^h 0^m P. M. and 5^h 10^m on October 16, indicate the maintainance of nearly normal relations between the standards compared. This date comes from the table which you have.

The statement here made is so greatly at variance with the ordinary view, that I have considered it important to verify it by a large number of special experiments, an account of which is necessarily omitted here. Any investigator can verify it by making continuous comparisons on stormy days on which a nearly constant temperature is maintained. It is essential, in these experiments, that the observer shall not touch the bars compared. It is also essential that the comparing room shall have dimensions of considerable extent, especially in height. The latter dimension should never be less than 12 or 15 feet.

(2) *When contact takes place through the hands between the person of the observer and the bars of metal compared, a nearly instantaneous increase in length takes place which diminishes in a slightly decreasing ratio as long as the contact is continued, but which ceases instantly when the contact is broken.*

The observations of September 29, Exhibit K, and of October 2 and 3, furnish an illustration of the facts, first, that the increments produced by successive charges of heat through contact between the hand and the three bars of metal are nearly equal; second, that there is neither an increase nor a decrease in length during the five minutes immediately following the charge; and, third, that the increase is nearly the same in amount for the steel, the bronze and the glass bars. The time required for the complete dissipation of the heat stored in the bars in this manner is nearly a constant, namely about 3 hours.

Many observations have been made, showing that equal increments in length correspond nearly with equal increments in time, at least for moderate intervals of time. Thus, in the observations of October 2 the total increase in length after four charges, made at intervals of five minutes, is the same as three times the increment in length due to the first charge.

Substantially the same results were obtained by placing a rectangular block of iron having a temperature of about 300° in contact with one of the vertical faces of the bar of metal. It should be noted that contact through the hands repeated at short intervals furnish a constant supply of heat only for a short time.

(3) *Whenever a change of temperature occurs, the effect of the change is indicated by thermometers which rest horizontally upon the upper surface of the bar of metal, in advance of the actual change in length.* It follows, therefore, that if the observations are reduced to an assumed higher temperature by means of the known coefficient of expansion, the computed length will be inevitably too short for a rising temperature, and too long for a falling temperature.

It is just here that we must look for an explanation of the anomalies which have given so much trouble. According to present indications metals *do not* "crawl" through changes in their molecular structure. For a rising temperature the change in length is not as rapid as the corresponding change in the indications of the thermometer with which the temperature of the metal is supposed to be measured. The strongest emphasis must be placed upon the fact that increments in length will *never* overtake those given by the thermometers, as long as the increase of temperature is continuous, even when it does not amount to more than one or two-tenths of a degree per hour.

(4) *The amount by which the indicated reading of the thermometer fails to indicate the real temperature of the standards compared, varies with their mass, and their shape, being less for*

a larger surface area, and greater for a large mass having equal dimensions in cross-section.

The last two columns in Exhibits K and L represent the relative lengths of the bronze bar R_2, the steel bar R_3 and the glass bar G, at 62°.0 Fahr., after the corrections for absolute error in length at this point have been applied. A slight examination will show that the systematic character of the relations given is very marked. They commence to be exhibited the moment a change of temperature is noted by thermometers Nos. 1 and 2, which rest horizontally upon the surface of R_2 and of G respectively. It will be seen, also, that the discordances between R_2 and G are much greater than those between R_2 and R_3, the mass of G being more than three times as great as that of R_3.

(5) *The systematic errors in question practically disappear whenever the readings of the thermometers placed upon the surface of the bars, remain in agreement with the indications of a thermometer supported in air at the same hight as the upper surface of the bars of metal for a period of three hours.*

In other words, these systematic deviations disappear when equilibrium occurs between the temperature of the upper surface of the bars compared, of the thermometers employed and of the surrounding air.

The determination of the exact time of equilibrinm is facilitated by the comparison between the two thermometers which rest upon the bronze and the glass bars. For a rising temperature the thermometer upon the glass bar reads from 0° to 1°.0 higher than that upon the bronze and the steel bars, while for a falling temperature it reads from 0° to 1°.0 lower than the other thermometers. The agreement between the readings of these thermometers is therefore an evidence that equilibrium has been reached. That this variation is not due to reflection from the smooth surface of the glass, is shown by the fact that the same phenomenon is seen when a glass bar having a sand-surface finish is substitutud for G. That the question of mass is not involved, is shown by the fact that there is no change in the observed relation when a glass bar having the same mass as R_2 is substituted for G. All indications point to the conclusion that the variation here noted is due to the greater absorptive and emissive power of the glass.

(6) *The danger of the introduction of systematic errors of the class indicated is much greater for slow than for rapid changes of temperature, and is also much greater when the direction of the flow changes from plus to minus than when it is continuous in the same direction.*

This conclusion is clearly confirmed by my experience with the new comparing room beneath the rotunda of the Observatory at Cambridge. It was found impossible to guage the delivery of the heat and the cold stored up in the immense masses of stone with which the comparing room was surrounded.

The comparing room of the International Bureau at Breteuil is for most purposes incomparably the best in the world, but it must be said that danger of the introduction of systematic errors is always present on account of the great length of time required to pass from one temperature to another, Even the immense mass of the comparator may itself become a reservoir of heat and cold. In fact, it seems to be almost impossible to avoid errors of this class for extremely slow changes of temperature. This danger relates only to comparisons between standards of very small and of very large mass. The present form of the prototype standards is so admirably adapted to take the temperature quickly that there is little danger in the direction indicated when standards of the same class are compared.

It is this slow but sure movement of the molecules of matter in the metal employed, a movement far too subtle to be studied in detail with the aid of the most delicate thermometer, but whose integration can be accurately measured with the aid of the microscope, which produces the apparent variations in length between *comparisons made at differene times of the day and especially at different seasons of the year.* The conditions under which equilibrium is determined vary with the rapidity of the increase or of the decrease of the temperature, and hence, they vary also with the season of the year. It hardly needs to be added, that these conditions must be determined for the particular location of the comparator with which the comparisons are made. At Waterville the time required for the equilibrium to be established is about three hours, but in the under-ground comparing room at Cambridge equilibrium was often delayed many hours, and in some cases, probably for several days after the indicated agreement between the temperature of the graduated surfaces of the bars and the surrounding air.

(7) *Admitting the observed fact that the thermometer placed upon the surface of the bars compared, always gives a wrong indication except at the moment of real equilibrium, it follows that the constant tendency of obsevers is to obtain for the length of a given standard at an assumed higher temperature, a result which is too small, since the comparisons are ordinarily made without forethought under a rising temperature.* The time between 10^h A. M. and 3^h P. M. is the most unfavorable which can be selected. So also the inevitable tendency is to make the derived length relatively shorter in summer than in winter.

The observations of November 3, exhibit L, furnish a good illustration of the effect of both slow and of rapid changes of temperature. During the night of November 2, the temperature of the comparing room and also of the room adjoining was kept at about 85°, until two o'clock on the morning of November 3, when it was allowed to diminish gradually. At the time of the first observation at 6^h 30^m, the computed relation between R_2 and R_3, made R_3 13 divisions of the micrometer shorter than R_2. Until ten o'clock when equilibrium was apparently established between R_2, R_3 and the surrounding air, the deviation gradually diminished in value when the normal relation between R_2 and R_3 was established. In the case of R_2 and G the deviation was much more rapid, but the normal relation was not reached until half an hour later. At $12^h\ 15^m$ the shutters of the four windows in the upper part of the partition between the two rooms were opened and a current of air having a temperature of about 95° was allowed to flow through until $1^h\ 50^m$ P. M. when the windows were closed. At about 5^h the surface equilibrium between R_2 and R_3 was established, while for R_2 and G it occurred near 6^h. During the night, under a slowly falling temperature, a complete reversal of the indications took place. The stationary period for R_2—R_3 was reached at about $11^h\ 30^m$, while for R_2—G it had not been reached at midnight.

(8) *The objection which will without doubt be made, that the temperature should be determined with thermometers placed in wells sunk in the bars of metal employed and surrounded with mercury, instead of being placed horizontally upon the upper surface, is not a valid one.*

The practice of placing thermometers in wells in a vertical position is indefensible, because the stems are exposed to contact with layers of air at different heights. There is less objection to the submergence of the thermometer in mercury at or near the centre of the bar, but unless the air is excluded from the groove, there will be very little gain in this arrangement. A thermometer mounted in this manner fails entirely, however, to indicate the outward flow of the heat from the surface. It is, indeed, true that under rapid changes of temperature the immersed thermometer will, at least at first, correspond more nearly with the internal temperature of the bar than the one which rests upon the surface, but for all slow changes, and especially when changes in the direction of the flow occur, the danger of the introduction of systematic errors is greater than for surface exposure.

According to present indications the real temperature of the entire mass of the metal will be obtained by taking the arithmetical mean of the readings of the temperature at the surface and at the center part of the bar of metal.

(9) *Whenever a constant temperature can be maintained for three and one-half hours, the point of complete saturation will be so nearly reached that the microscope will fail to detect variations in the length of the standards compared.*

Of course this remark relates to bars of metal which have considerable mass. For thin bars the time required is much less. When eight or ten comparisons are made under this condition, the result obtained will be found to have as much value as a much larger number of comparisons made at equi-distant intervals from the critical period of quiesence of temperature. The mean of the results of two or three day's work under these favorable conditions will give a near approximation to the real relation between the standards compared.

(10) *The apparent stationary point as indicated by the reading of the surface thermometer can be regarded as the real point of equilibrium only when the thermometric indications have remained constant for at least three hours.* In other words, we must know the temperature conditions which have existed during the three hours immediately preceding before we can assume that the real point of equilibrium has been reached. The observance of this precaution is of the utmost importance, since for a rising temperature it will be inevitable that a wrong relation between the standards will be determined for every observation between the time when the reading of the surface thermometer becomes stationary and the time elapsed before the two standards reach the normal relation for that constant temperature. In a general way the reverse action takes place when the temperature is derived from the thermometer placed at the center of the standard bar. If the temperature is determined by taking the mean of the thermometer readings at the surface and at the interior of the the bar, a near approximation to the mean temperature will be obtained.

(11) *The error in the determination of the normal relation between the standards, due to the more sluggish action in the case of the metals, will not be elimnated as long as there is a movement of the temperature in the same direction if either the surface or the interior temperature be taken alone, but will be nearly eliminated when the mean of the two indications is taken.*

On the other hand, the elimination will take place, more or less perfectly, in the combination of observations, having nearly equal upward and downward drifts of temperature. It seems to be more exactly secured in those cases in which there is a gradual rise or fall from equilibrium. More observations are needed to determine the law under which the elimination takes place. This elimination will ordinarily be more perfect for surface indications than for those at the interior of the standard bar.

(12) *After every precaution has been taken, instances sometimes occur in which an entire series of observations yield results which differ persistently from the normal relation.*

The amount of this deviation rarely exceeds 1μ. It is possible that the explanation of this result may be found in the conclusion given under division (11), but I have been thus far inclined to attribute it to variations in the humidity of the atmosphere.

(13) *The time required to pass from complete saturation at one temperature to complete saturation at another temperature is nearly a constant.*

In the case of the three metals under consideration this time does not vary much from three and one-half hours, the time for the glass bar being a little less than that for the bars of bronze and steel. This conclusion is verified by all of the thirteen cases given under exhibit M.

The accurate determination of this time will be possible only when the means are at hand for a better control of the temperature than are at present available. The effect of a change of the temperature of the surrounding air in the same direction as that which is taking place in the bar under observation will be apparently to shorten the time required for complete saturation, while that in the opposite direction will apparently lengthen the time.

The time required to pass from complete saturation at 0° to complete saturation at 1° is nearly the same as the time required to pass from complete saturation between the limits 0° and 100°.

This law has been found to hold nearly true as far as $T=30°$, and it probably holds approximately true for any value of T whatever.

The law may be stated in another form as follows: Two bars of metal are allowed to remain in a room having a given constant temperature until complete saturation has taken place. One of them is removed to a room in which a temperature, one degree higher than in the first room, is maintained. The other is removed to a room in which a temperature e. g. 30° higher is maintained. Then each bar will take the temperature of the medium in which it is placed in the same time, viz. in about three and one-half hours.

(15) *It follows as a consequence of the law given under division (14) that the relative specific heat of metals may be obtained by comparing the times required for complete saturation if the bars of metal compared have the same mass and the same shape, since the change in length is a measure of the total amount of the work done by the heat which has been absorbed by the metal.*

(16) *The law under which changes of length take place in bars of metal under the action of a constant supply of heat may be expressed by a series under "Taylors Theorem" in terms of the ascending powers of the time.*

Let $F =$ the total change in length between the limits t_0 and t.

Let $F_0 =$ the correction in length at the instant t_0.

Let $\frac{d\,f}{d\,t} =$ the rate of change during the instant d t.

Then $F=F_0+\frac{d\,f}{d\,t}t+\frac{1}{2}\frac{d^2\,f}{d\,t^2}t^2+\frac{1}{6}\frac{d^3f}{dt^3}t^3+\frac{1}{24}\frac{d^4f}{dt^4}t^4+\&c.$

The values of $\frac{d\,f}{d\,t}$, $\frac{d^2f}{dt^2}$, $\frac{d^3f}{dt^3}$, &c. can be determined from the series of differences between the changes in length which occur in any assumed equal intervals of time by means of the following formulas. [See paper by Miss Anna Winlock, in the Memoirs of the American Academy of Arts and Sciences, Vol. XI, Part IX, page 248.]

Let w=any interval of time whatever.

Let $\Delta_1\ \Delta_2\ \Delta_3$ &c.=the successive orders of differences in the given series of comparisons.

Let $\Delta^{i}\ \Delta^{ii}\ \Delta^{iii}$ &c.=the differences opposite the initial functions.

$$\text{Then } \Delta_1 = \frac{\Delta_1^{+\frac{1}{2}} + \Delta_1^{-\frac{1}{2}}}{2},\quad \Delta^{III} = \frac{\Delta_3^{+\frac{1}{2}} + \Delta_3^{-\frac{1}{2}}\ \text{\&c.}}{2}$$

$$\Delta^{II} = \Delta_2 \qquad \Delta^{III} = \Delta_3, \text{ \&c.}$$

$$\frac{d\,f}{d\,t} = \frac{I}{w}\left[\Delta^{I} - \tfrac{1}{6}\,\Delta^{III} + \tfrac{1}{30}\,\Delta^{V} - \tfrac{1}{140}\,\Delta^{VII} + \tfrac{1}{630}\,\Delta^{IX} - \text{\&c.}\right]$$

$$\frac{d^2f}{dt^2} = \frac{I}{w}\left[\Delta^{II} - \tfrac{1}{12}\,\Delta^{IV} + \tfrac{1}{90}\,\Delta^{VI} - \tfrac{1}{560}\,\Delta^{VIII} + \tfrac{1}{3150}\,\Delta^{X} - \text{\&c.}\right]$$

$$\frac{d^3f}{dt^3} = \frac{I}{w^3}\left[\Delta^{III} - \tfrac{1}{4}\,\Delta^{V} + \tfrac{7}{120}\Delta^{VII} - \tfrac{41}{3024}\Delta^{IX} + \text{\&c.}\right]$$

$$\frac{d^4f}{dt^4} = \frac{I}{w^4}\left[\Delta^{IV} - \tfrac{1}{6}\,\Delta^{VI} + \tfrac{7}{240}\Delta^{VIII} - \tfrac{41}{7560}\Delta^{X} + \text{\&c.}\right]$$

$$\frac{d^5f}{dt^5} = \frac{I}{w^5}\left[\Delta^{V} - \tfrac{1}{3}\,\Delta^{VII} + \tfrac{13}{144}\Delta^{IX} - \text{\&c.}\right]$$

$$\frac{d^6f}{dt^6} = \frac{I}{w^6}\left[\Delta^{VI} - \tfrac{1}{4}\,\Delta^{VIII} + \tfrac{13}{240}\Delta^{X} - \text{\&c.}\right]$$

$$\frac{d^7f}{dt^7} = \frac{I}{w^7}\left[\Delta^{VII} - \tfrac{5}{12}\,\Delta^{IX} + \text{\&c.}\right]$$

$$\frac{d^8f}{dt^8} = \frac{I}{w^8}\left[\Delta^{VIII} - \tfrac{1}{3}\,\Delta^{X} + \text{\&c.}\right]$$

$$\frac{d^9f}{dt^9} = \frac{I}{w^9}\left[\Delta^{IX} - \text{\&c.}\right]$$

$$\frac{d^{10}f}{dt^{10}} = \frac{I}{w^{10}}\left[\Delta^{X} - \text{\&c.}\right]$$

The following numerical equations result from the data given in experiments 1 to 12 under exhibit M. It will be understood that they represent only an approximation to the true values of the coefficients on account of the imperfect character of the data, due to the partial failure to maintain a constant temperature during the comparisons. They will, however, serve the present purpose as examples of this method of proceedure.

The values of the coefficients, which are expressed in the form of logarithms, are derived from the differences opposite the middle point of each series, viz: along the line of the dashes. For a wide range of temperature, it was found necessary to extend the observations sufficiently far to make the intervals equal to 48 minutes.

No. of Experiments.	Name of Standard.	Range of Temperature.	Equations.						
1	R_2	34.05° to 64.1°	$F = -88.7$ div.	$-9.95646t$	$+7.51600t^2$	$-5.22540t^3$	$+3.26712t^4$	$-9.89465t^5$	$-8.46347t^6$
2	R_2	32.05 to 50.1	$F = -27.3$	$-9.63383t$	$+7.39924t^2$	$-4.92046t^3$	$+2.84584t^4$	$-0.45414t^5$	$+6.65729t^6$
3	R_2	59.05 to 56.8	$F = +43.2$	$+1.12985t$	$-1.42516t^2$	$+1.01019t^3$	$+1.54946t^4$	$-0.76882t^5$	$-1.33984t^6$
4	R_2	68.0 to 64.1	$F = -165.2$	$+8.74688t$	$+6.22015t^2$	$+5.28043t^3$	$-3.56767t^4$	$-0.54120t^5$	$-9.21214t^6$
5	R_3	34.05 to 64.1	$F = -116.5$	$-9.64849t$	$+7.29454t^2$	$-5.05536t^3$	$+3.02783t^4$	$-0.02650t^5$	$-8.05955t^6$
6	R_3	32.05 to 49.7	$F = -126.7$	$-9.40041t$	$+7.12375t^2$	$-5.00743t^3$	$+3.93092t^4$	$-9.41771t^5$	$-8.12711t^6$
7	R_3	59.05 to 56.6	$F = -2.4$	$+9.23430t$	$-7.04299t^2$	$-5.21995t^3$	$+2.51125t^4$	$+1.21789t^5$	$-8.50971t^6$
8	R^3	68.0 to 64.1	$F = -158.4$	$+8.72657t$	$+6.52890t^2$	$+4.64216t^3$	$-3.47905t^4$	$+9.57116t^5$	$+9.26711t^6$
9	G	34.05 to 63.6	$F = -126.7$	$-9.22912t$	$+7.27820t^2$	$-5.05174t^3$	$+2.47079t^4$	$-0.43310t^5$	$+8.23707t^6$
10	G	35.5 to 50.1	$F = -143.4$	$-9.25540t$	$+7.35942t^2$	$-5.50141t^3$	$+3.39968t^4$	$+9.57116t^5$	$-8.74524t^6$
11	G	68.0 to 63.0	$F = -135.4$	$+8.88843t$	$-7.25037t^2$	$+4.56674t^3$	$+3.09055t^4$	$+0.09404t^5$	$-8.79299t^6$
12	G	32.0 to 65.0	$F = +251.7$	$+9.68590t$	$-7.64267t^2$	$+5.09645t^3$	$-3.39461t^4$	$+1.55524t^5$	$-9.05920t^6$

That these eqations represent with a close approximation all the observations of a given series is proved by the agreement of the computed with the observed values of F which were not employed in the formation of the equations. The extent of the agreement is shown in the following table. The residuals (c—o) are not given from the data represented by the equations since these results must, from the nature of the solution of the problem, be zero, as in fact they are:

Experiment 1.

Time h m	(c—o)
7 51	+4.9 div.
8 7	+1.8
8 39	+1.3
8 55	+0.0
9 27	—0.8
9 43	—1.2
10 15	+0.5
10 31	+1.9
11 3	—2.0
11 41	+0.7

Experiment 2.

Time h m	(c—o)
10 16	—0.1 div.
10 32	+0.0
11 4	—0.6
11 20	—0.4
11 52	—1.1
12 8	—1.8
12 40	—2.6
12 58	+0.2
1 30	+1.5
1 46	+1.5

Experiment 5.

Time h m	(c—o)
7 55	+2.1 div.
8 11	+0.5
8 43	+0.7
8 59	+2.1
9 31	—0.9
9 47	—1.4
10 19	—0.8
10 35	+0.9
11 7	+2.0
11 23	+0.6

Experiment 6.

Time h m	(c—o)
10 6	+1.3 div
10 22	—0.3
10 54	+1.5
11 10	+1.9
11 42	—1.9
11 58	—1.6
12 30	+0.0
12 46	—0.1
1 18	+0.9
1 34	+2.2

Experiment 9.

Time h m	(c—o)
7 47	+3.6 div.
8 3	+3.0
8 35	—0.9
8 51	—0.6
9 23	—1.1
9 39	—0.9
10 11	+0.9
10 27	—0.6
10 59	+1.1
11 15	+0.5
12 45	—0.4
1 1	—0.7

Experiment 3.		Experiment 7.		Experiment 10.		Experiment 12.	
Time	(c—o)	Time	(c—o)	Time	(c—o)	Time	(c—o)
h m		h m		h m		h m	
2 31	—1.0 div.	2 24	—1.8 div.	2 52	+1.5 div.	10 12	+4.5 div.
3 3	+0.4	2 56	—0.9	3 22	+0.1	10 44	—0.2
3 35	+0.4	3 28	+0.3	3 56	—1.3	11 16	+0.9
4 7	+0.1	4 0	+0.7	4 26	+2.5	11 48	+0.1
4 39	—1.0	4 32	+0.8	4 58	+0.5	12 20	+0.9
5 11	+1.3	5 4	—3.1			12 52	—0.9

Experiment 4.		Experiment 8.		Experiment 11.	
Time	(c—o)	Time	(c—o)	Time	(c—o)
h m		h m		h m	
2 57	+0.0 div.	3 1	+0.3 div.	2 53	+0.9 div.
3 29	—0.3	3 33	+0.1	3 25	+0.5

It will be seen, therefore, that *all* of the equations are verified within the limits of the accidental errors of observation.

We have a second test in the agreement of the value of the observed change of length between the time when the standard was brought into the comparing room and at the conclusion of the series, with the computed value of the change obtained by multiplying the coefficient of expansion of the standard under observation by the difference of the temperature at the beginning and the end of the series. The computed value of the relation between the length of the yard and the distance between the stops obtained from the equation at the initial time is, however, subject to much uncertainty in these observations, since in the removal of the standards it was necessary to carry them a considerable distance through two intervening rooms. The rapidity of the change at the initial time of observation will also prevent a very accurate determination of the length at that instant, neither can the computation from the equation be extended to a time much earlier than that of the first comparison. We may take experiment 4 as an example. The computed value of F at 9^h 27^m is 220.4 div. The total change in length during the interval of $3^h\ 27^m$ was 269.6 div. The absolute coefficient of standard R_2 is 16.83 divisions for each degree Fahr. Hence the total change equals [16.83 div.] $[50^{\circ}.10—32^{\circ}.05]=303.8$ div. The difference, 34.2 div., is perhaps as small as one should expect in the present case.

It is frequently desirable to obtain an expression for the reduction from one point of time to another in terms of the first power of the time. For this purpose we may make use of the form of development suggested by Struve.

Let $\frac{df^1}{dt}\ \frac{df^2}{dt}\ \frac{df^3}{dt}$ &c. represent the variations for one miuute at equal intervals of time, e. g. 16 minutes. We shall then have:

$$F=F_o+\frac{1}{90}\left(\frac{7df^1}{dt}+\frac{32df^2}{dt}+\frac{12df^3}{dt}+\frac{32df^4}{dt}+\frac{7df^5}{dt}\right)t$$

The successive values of $\frac{df^1}{dt}\ \frac{df^2}{dt}$ &c. can be found by taking the difference between the computed values of $\frac{df}{dt}$ for the times $\overset{-\frac{1}{2}}{T}$ and $\overset{+\frac{1}{2}}{T}$.

Since considerable time must be allowed for the requisite adjustment after the standards have been removed to the comparator, we must rely upon the value of F computed from the equation at the instant when the standard was removed to the comparing room, but we cannot, unfortunately, make use of the observed data until after the exposure has been continued for at least three hours, since the thermometer never indicates the real temperature of the bar of metal while changes of temperature are in progress. Hence, if the latter formula is employed we must make the equal intervals large enough to reach a point of time near the end of the series, but the development will fail if the entire interval is too great.

In order to illustrate the application of this formula, we make use of the computed values of F derived from equation 12 of exhibit M. The interval chosen is 8 minutes.

Time h m	F	$\frac{d'f}{dt}$
9 41.5	+ 10.623 div.	
9 42.0	14.106	+6.967 div.
9 42.5	17.590	
9 49.4	+ 61.056	
9 40.0	63.836	+5.559
9 50.5	66.615	
9 57.5	+101.103	
9 58.0	103.286	+4.371
9 58.5	105.474	
10 5.5	+132.724	
10 6.0	134.452	+3.457
10 6.5	136.181	
10 13.5	+157.721	
10 14.0	159.086	+2.733
10 14.5	160.432	

We have, therefore, for the reduction from $9^h\ 42^m$ to $10^h\ 14^m$ $F=\frac{32}{90}\left[7\times6.967+32\times5.559+12\times4.371+32\times3{,}457+7\times2.733\right]=$ 145.4 div. The variation in length between these limits of time, derived from the differential equation, is equal to [159.1—14.1]=145.0 div.

It will not escape attention that the equations given on page 69 represent only the given series of observations from which they were derived. For example, the equation derived from the comparisons of experiment 1 will hold for any point of time during an interval of five hours and for any temperature between 34° and 64°, if we start at 34°, but it will not necessarily hold if we start, for example, at 35° and go to 65°. Within the short time since this line of investigation was undertaken no satisfactory analytical expression has been found from which the length of a given standard at any assumed time and for any given temperature can be determined.

(17) *Under variations of temperature not exceeding two degrees per hour the maximum error possible arising from the failure of the bar of metal to follow the indication of the thermometer placed upon its upper surface, is not far from 4 mikrons in the determination of the relative lengths of the bronze and the steel bars and not far from 10 mikrons in the case of the bronze and the glass bars.*

If the thermometer is placed at the center of the mass of the bar, an error of nearly the same amount but with the opposite sign, will be introduced. This law seems to hold true in the case of either a slow or a rapid change of temperature, but further observations are needed to determine the law in passing, at the stationary point, from a rising to a falling temperature and *vice versa.*

(18) *The limit of the maximum error, stated under the previous division, varies with the mass of the material of which the thermometers are constructed.*

The diameter of the stem and the mass of the metal backing which sometimes forms the support of the thermometer are the two elemnnts which have the greatest influence.

(19) *Any thermometer employed to measure the temperature of the standard bars compared, must have its errors determined under the same conditions as those under which the standard thermometer is exposed.*

Hence, the errors of secondary thermometers must be determined for a horizontal position and for contact with the same piece of metal as the primary standard.

The Yale Centigrade thermometer Y_{62} has been adopted as the primary standard in this investigation, and whenever any other thermometer has been employed its errors have been determined by comparison with Y_{62}.

(20) *Every thermometer employed to measure the temperature has its individual idiosyncrasies which must be studied in detail independently of its normal relations to the primary standard.*

These peculiarities depend, in a general way, upon the form and the mass of the thermometers. As an illustration of the necessity of this precaution, it may be mentioned that certain thermometers of Series I invariably give wrong indications under certain conditions, the errors of the same point determined at equilibrinm being the standard of comparison. So also, the relations determined from day and from night observations are for some thermometers quite different from the relations which exist when the metal backing is removed and only the glass tube is used.

We have a test of the accuracy of the results obtained under exhibits K and L, in their accordance with the results obtained in previous years. In the following table are given the values of R_2—R_3 and of R_2—G_a, both for the yard adn the meter. In the case of the yard these values have been corrected by the addition of 9.8 div. for R_2—R_3 and of 1.1 div. for R_2—G_a. In the case of the meter the constants have not been taken out.

For the Yard.

Date.	At 62°.0 R_2—R_3	No. Obs.	At 62°.0 R_2—G_a	No. Obs.
Sept. 25,	—2.30 div.	2	—2.80 div.	2
26,	+5.20	2	+7.10	2
27,	—2.50	2	+7.95	2
28,	—0.89	7	—2.90	7
29,	—0,70	15	+3.94	17
30,	—2.59	7	+1.17	7
Oct. 2,	+0.50	3	—2.03	3
3,	—0.12	7	+3.14	7
4,	—1.70	7	+2.91	7
5,	—1.76	8	—1.01	8
6,	—0.17	3	+5.83	3
Means	—0.97		+1.18	

For the Meter.

Date.	At 61°.0 R_2—R_3	No. Obs.	At 62°.0 R_2—G	No. Obs.
Oct. 10,	—2.05 div.	2	+15.45 div.	2
11,	—3.69	7	+11.34	7
12,	—3.20	7	+ 8.63	7
13,	—3.62	6	+16.05	6
14,	—3.20	2	+19.05	2
16,	—4.20	5	+18.16	5
17,	—4.40	2	+23.10	2
18,	—10.3	1	+ 8.1	1
25,	—6.60	4	——	–
27,	—6.76	14	——	–
Means	—4.84		+14.04	

For the yard, therefore, the deviation from the result given by the former series does not in either case much exceed half a mikron. For the meter we have, after the reduction for the horizontal curvature is applied, the following values for the corrections to be applied to R_3 and to G_a:

Correction to R_3 from the present series = $+3.5\mu$.

Correction to R_3 from the observations of 1884-5 = $+2.8\mu$.

Correction to G_1 from the present series = $+24.8\mu$.

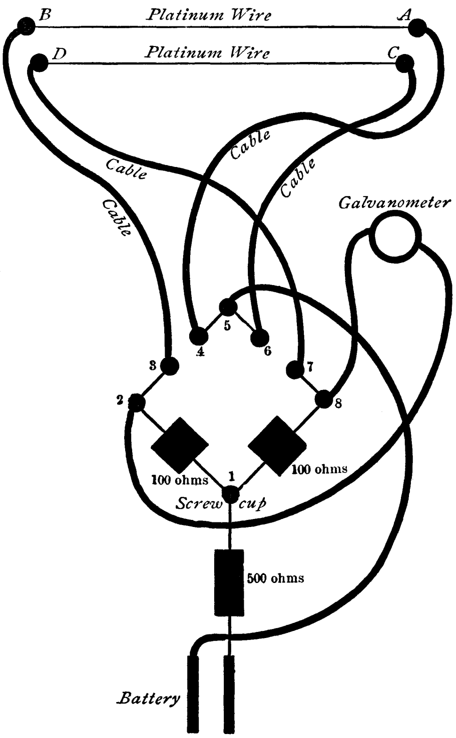
Correction to G_1 from the observations of 1884-5 = $+21.4\mu$

It is probable that the first published value of the correction to G_a, viz. $+21.4\mu$ is erroneous, notwithstanding the close agreement between the early and the later values for the yard, since the necessity of making the comparisons only at times about three hours subsequent to an apparent equilibrium of temperature between the standards and the surrounding air, was not at that time recognized.

In conclusion, attention is again called to the fact that this investigation relates only to the constancy of the relative coefficients of expansion between these particular bars of metal and only between the limits of the temperatures at which actual observations have been made.

It may well be doubted whether the attempt to limit the comparisons to the imposed condition of molecular rest has been successful in this series of experiments, but the writer is confident that a near approximation to the exact relation between the standards R_2 and R_3 has been obtained. It is certainly true that under the conditions imposed a constant value of R_2—R_3 at 62°.0 Fahr. will be obtained from observations at any temperature between 0° and 95°.

Every indication points to the conclusion that further advance in metrological investigation must be along the line indicated in this paper. The writer is now engaged in experiments with a modified form of Langley's bolometer which gives promise of most interesting and valuable results. The full account of the method will be reserved for a subsequent paper, but the general outline of the method will be seen from the following figure and the accompanying description.



The wire AB is insulated at the center of the standard whose temperature is desired. CD is another wire stretched along and very near the upper surface, also insulated from the bar. The galvanometer will then indicate the difference between the temperature of the upper and the lower wires, and with a close approximation between that of the surface and that of the central part of the bar. Eventually, probably a third wire will be stretched along the lower face of the standard which will form a pair with a second line through the central part, the connections being made in such a manner as to allow the necessary changes in connections to be made at the galvanometer.

The practical outcome of this investigation may be stated as follows:

(*a*) The normal dimensions of matter are obtained from direct observations only when it is in a state of approximate molecular rest.

(*b*) In the comparison of standards of length the best results will be obtained by delaying the observations until the standard bars of metal have remained for three and one-half hours at a temperature which at no time during this interval has changed more than two or three tenths of a degree.

(*c*) In the comparison of thermometers the best results will be obtained by delaying the observations until they have remained at a constant temperature for twenty minutes under the condition of open air exposure, while for the condition of contact with the surface of metals the indication will be largely controlled by the temperature of the metal.

SERIES I.—July 8, 1887.

EXHIBIT (A).

Temperature of mercury and of water after removal from melting ice to a nearly constant air temperature of 82° Fahr.

Number of vessel.	Material.	Hight.	Diameter.	Hight of liquid.	No. of cubic Centimeters of liquid.	Liquid.
1	Glass.	13 in.	2 in.	2.4 in.	100	Mercury and Water.
2	"	6.1 "	2¼ "	4.8 "	300	Mercury.
3	"	10.0 "	10 "	9.3 "	100	Water.
4	Porcelain.	....	....	....	100	Mercury.

Reading of Thermometers. Exposure at 10^{h} 5^{m}. No. 2 removed in advance of Nos. 1, 3, and 5.

Time.	Stationary Thermometer.	No. 1. Mercury.		No. 1. Water.		No. 2.		No. 3.		No. 4.
		Bottom.	Top.	Bottom.	Top.	Bottom.	Top.	Bottom.	Top.	
h m	°	°		°		°		°		°
10 10	81.2	39.0	....	37.5	....	43.6	....	43.6	....	51.0
10 15	81.0	45.8	....	44.0	....	49.6	....	50.0	....	58.8
10 20	80.8	49.9	....	48.8	....	53.6	....	54.1	....	62.4
10 25	81.0	52.8	....	52.0	....	56.1	....	57.2	....	65.0
10 30	81.2	55.9	55.5	55.6	56.2	59.6	60.8	60.1	64.0	67.7
10 35	81.2	59.0	59.1	59.1	59.7	62.2	63.7	63.8	67.0	70.9
10 45	81.3	63.0	62.9	63.0	63.8	64.9	66.9	67.0	69.8	72.0
10 55	81.5	66.0	65.9	66.0	66.2	68.1	69.0	69.6	71.6	73.7
11 05	81.8	68.5	68.7	68.9	69.0	70.8	71.4	72.0	74.0	74.9
11 15	81.7	70.2	70.4	70.7	70.8	72.4	73.1	73.8	75.5	75.8
11 25	81.8	72.4	72.6	72.6	72.9	74.2	74.9	75.9	76.4	77.1

Reading of Thermometers.—*Continued.*

Time.	Stationary Thermometer.	No. 1. Mercury.		No. 1. Water.		No. 2.		No. 3.		No, 4.
		Bottom.	Top.	Bottom.	Top.	Bottom.	Top,	Bottom.	Top.	
h m										
11 35	81.8	73.4	73.5	73.5	73.7	75.1	75.7	76.9	77.8	76.9
11 50	81.9	75.1	75.2	75.3	75.2	76.7	77.0	78.1	79.0	77.1
12 05	82.0	76.2	76.3	76.3	76.3	77.4	77.9	78.8	79.5	77.4
12 30	82.1	78.2	78.1	78.3	78.5	78.0	79.2	80.0	80.9	77.8
2 45	82.1	80.6	80.7	80.7	80.7	80.8	80.8	81.7	82.0	78.6
3 00	82.2	81.0	81.0	81.0	81.0	80.9	80.8	81.7	82.0	78.7
3 10	Lowered position of every vessel 10 inches.									
3 45	82.2	81.5	81.4	81.5	81.5	81.2	81.2	81.7	82.0	79.0
3 50	Raised every vessel 6 inches above first position.									
4 00	82.4	81.8	81.8	81.8	81.8	81.6	81.6	82.1	82.1	79.2
4 15	82.4	82.0	81.9	81.9	81.9	81.7	81.6	82.1	82.1	79.5
4 30	82.4	82.0	82.0	81.9	81.9	81.7	81.7	82.1	82.1	79.7
4 35	Lowered every vessel to the first position.									
4 40	82.5	82.0	82.0	....	....	81.7	81.7	82.2	82.3	80.0
5 15	82.5	82.0	82.0	....	....	81.8	81.8	82.3	82.3	80.1
5 30	82.4	82.1	82.0	....	....	81.8	81.8	82.3	82.3	80.1
7 00	82.2	82.0	82.0	....	....	81.8	81.8	82.2	82.2	80.5

SERIES II.—July 14th and 15th.

Temperatures of mercury and of water exposed in tin vessels.

Removed from melting ice at 8h 45m

			Mercury				Water				
No. inches in diameter,		2	2	2	3	6	2	2	2	3	6
No. cubic centimeters,		50	100	150	100	100	50	100	150	100	100
Time. h m	Air Temperatures.	°	°	°	°	°	°	°	°	°	°
July 14.— 8 55	65.73	56.0	55.0	55.1	54.6	53.4	42.8	39.7	37.1	43.0	46.8
9 05	65.80	61.8	60.4	60.4	60.1	59.2	50.0	46.0	42.9	50.0	53.2
9 20	65.80	63.2	62.6	62.6	62.2	62.0	54.2	50.7	49.8	54.3	57.0
9 30	65.95	64.1	63.3	63.3	63.3	63.3	58.2	54.6	52.7	57.2	59.7
10 10	66.03	65.0	64.7	64.8	65.1	65.0	61.4	59.4	59.5	62.0	63.0
10 30	66.10	65.1	64.7	64.8	65.0	64.9	63.1	62.1	61.5	63.2	64.3
10 40	66.27	64.9	64.8	64.8	65.0	65.0	63.9	62.6	62.0	63.5	64.1
12 00	65.93	65.0	65.0	65.0	65.1	65.2	64.9	64.6	64.5	65.0	65.3
12 20	66.15	65.1	65.1	65.0	65.2	65.3	64.9	64.8	64.7	65.1	65.2
1 30	66.06	65.2	65.3	65.2	65.7	65.6	65.0	65.0	64.9	65.2	65.4
3 30	66.31	65.2	64.9	64.8	65.2	65.7	66.0	65.8	65.7	66.0	66.0
5 47	66.69	66.2	66.0	66.0	66.0	66.0	65.8	65.2	65.2	65.7	66.0
6 40	66.78	66.2	66.1	66.2	66.2	66.1	65.9	65.3	65.2	66.0	66.1

SERIES II.—July 14th and 15th.—*Continued.*

			Mercury					Water				
No. inches in diameter,			2	2	2	3	6	2	2	2	3	6
No. cubic centimeters,			50	100	150	100	100	50	100	150	100	100
	Time. h m	Air Temperatures.										
July 15th.	5 00	65.90	65°.9	65°.9	65°.8	65°.7	66°.0	65°.9	65°.9	65°.9	65°.1	65°.3
	5 10	—Removed from comparing room to melting ice.—										
	5 13	.....	55.0	53.0	52.0	50.7	48.1	56.1	52.1	51.8	49.9	53.9
	5 18	.....	45.5	43.3	42.9	42.9	41.0	50.5	46.6	46.8	45.8	48.2
	5 23	.....	41.1	40.0	39.3	39.1	40.2	44.3	42.1	42.8	41.2	45.2
	5 28	.....	38.8	37.9	37.2	37.1	38.7	42.0	40.0	40.0	38.7	42.6
	5 33	.....	37.5	36.8	36.2	36.1	38.1	39.4	37.8	37.5	36.0	42.0
	5 38	.....	36.0	36.1	35.8	35.6	38.2	38.1	37.4	35.8	35.8	39.8
	5 43	.....	36.0	35.5	34.9	35.2	37.8	37.0	36.4	35.1	35.8	39.1
	6 20	.....	35.6	35.3	35.0	34.7	37.2	34.2	34.2	34.8	34.8	37.3
	6 25	.....	35.5	35.0	34.9	34.5	37.0	34.0	34.4	34.8	34.6	37.4
	7 10	66.0	35.8	36.1	35.2	34.7	37.0	33.2	34.2	33.9	34.1	37.2

EXHIBIT (B).

Observed relations between the bronze and steel standards of length.

METERS.

Date of Comparisons.	Comparison with	Equations between R_3 and A_0.	Weights.
1884, Feb. 5 to Mar. 11.	R_2 (meter)	$R_3+4.0\mu=A_0$	1
“ “ “	R_2 (yard)	$R_3+4.1\mu=A_0$	1
“ “ “	R_1 (meter)	$R_3+2.4\mu=A_0$	1
“ “ “	R_1 (yard)	$R_3+2.7\mu=A_0$	1
“ Nov. 16 to Dec. 21.	R_2	$R_3-1.2\mu=A_0$	½
“ Dec. 21 to Dec. 31.	R_2	$R_3+2.2\mu=A_0$	2
1885. Jan. 2 to Feb. 15.	R_3	$R_3+1.9\mu=A_0$	3

Adopt. $R_3+2.4\mu=A_0$.

YARDS.

1884. Feb. 5 to Mar. 11.	R_3 (yard)	$R_3+1.0\mu=Y$	1
“ “ “	R_2 (meter)	$R_3-1.4\mu=Y$	1
“ “ “	R_1 (yard)	$R_3-0.6\mu=Y$	1
“ “ “	R_1 (meter)	$R_3-0.8\mu=Y$	1
1885, Jan. 12 to Feb. 16.	R_2	$R_3-0.5\mu=Y$	1

Adopt. $A_3-0.5\mu=Y$.

METERS.

Date of Comparisons.	Comparison with	Equations between A_4 and A_0.	Weights.
1884, Feb. 5 to Mar. 11.	R_2	$A_4+4.0\mu=A_0$	2
“ Mar. 12 to Mar. 21.	R_2	$A_4+1.3\mu=A_0$	2
“ Feb. 5 to Mar. 5.	R_1 (meter)	$A_4-0.4\mu=A_0$	1
“ “ “	R_1 (yard)	$A_4+0.0\mu=A_0$	1
1885, Jan. 10 to Feb. 15.	R_2	$A_4-0.9\mu=A_0$	3

Adopt. $A_4+0.2\mu=A_0$.

YARDS.

1884, Feb. 5 to Mar. 11.	R_2	$A_4+1.8\mu=Y$	2
“ Mar. 12 to Mar. 27.	R_2	$A_3+1.3\mu=Y$	2
“ Feb. 5 to Mar. 5.	R_1 (yard)	$A_4+1.4\mu=Y$	1
“ “ “	R_1 (meter)	$A_4+0.2\mu=Y$	1
1885, Jan. 12 to Feb. 24.	R_2	$A_4+1.8\mu=Y$	3

Adopt. $A_4+1.1\mu=Y$.

EXHIBIT C.

Relative lengths of steel and silver strips corresponding to variations of temperature between the limits +18° C. and +39° C.

Tension of strips=7 lbs.

I=increasing temperature.
D=falling temperature.
S=stationary temperature.
Δ=excess of micrometer reading of silver over steel.
1°=15.0 divisions of micrometer.

Date.	Y_{63}	$28^\circ.00-Y_{63}$	Δ	Δ reduced to 28°.00	Number of Comparisons.	Direction of Temperature.
1887.						
July 4.	29°.88	— 1°.88	+ 89.6 div.	+61.4 div.	5	D
" 4.	29.48	— 1.48	+ 82.7	+60.5	2	I
" 5.	27.92	+ 0.08	+ 54.9	+53.7	4	S
" 5.	28.92	— 0.92	+ 47.2	+61.0	2	S
" 6.	23.97	+ 4.03	+ 0.0	+60.4	3	S
" 8.	28.21	— 0.21	+ 51.0	+54.2	2	I
" 8.	28.08	— 0.08	+ 49.0	+50.2	3	D
" 9.	27.64	+ 0.36	+ 62.0	+67.4	8	I
" 10.	18.60	+ 9.40	— 69.3	+71.7	1	D
" 11.	18.13	+ 9.87	— 72.9	+75.1	3	I
" 12.	25.25	+ 2.75	+ 24.5	+65.7	4	D
" 12.	29.00	— 1.00	+ 59.4	+44.4	2	D
" 13.	24.65	+ 3.35	+ 1.3	+51.5	6	D
" 13.	29.10	+ 1.10	+ 89.3	+72.8	3	I
" 14.	20.50	+ 7.50	— 61.8	+50.7	2	D
" 14.	38.57	—10.57	+215.7	+57.2	3	D
" 14.	37.75	— 9.75	+210.7	+64.5	7	I
" 14.	37.88	— 9.88	+211.2	+63.0	8	D
" 15.	21.39	+ 6.61	— 31.7	+67.4	7	I
" 16.	20.34	+ 7.66	— 47.4	+67.5	1	I
" 16.	20.27	+ 7.73	— 60.4	+55.5	8	D
" 18.	19.17	+ 8.83	— 68.4	+64.0	1	I
" 18.	19.73	+ 8.27	— 68.2	+55.8	3	S

EXHIBIT C.—*Continued.*

Equations of condition from groups of five observations each. All the observations are assumed to have equal weights.

	$(28^\circ.00-T)\Delta b$	a	Normal Equations.
$+64.4$ div.	$=a+8.81\Delta b$	$+63.3$	$+303.4=+\ 5a+\ 5.71\Delta b$
59.5	$=a+5.43$	$+58.9$	$+365.8=+5.71a+172.5\Delta b$
58.3	$=a+0.58$	$+58.2$	
59.7	$=a-1.12$	$+59.8$	$\Delta b=+\ 0.12$
61.5	$=a-7.99$	$+62.5$	$a=+60.6$

The coefficient between steel and the silver is therefore 15.12 div.

Hence 1 div. $=0^\circ.066$ C.

Residuals in divisions.	Residuals in terms of 1 degree C.
$+2.6$ div.	$+0^\circ.17$
-1.6	-0.11
-2.3	-0.15
-0.7	-0.05
$+2.0$	$+0.13$

EXHIBIT D.

Comparison of thermometers Y_{62} and G_2, which rest upon the surface of a large mass of iron with G_1, which is supported in air at nearly the same height as Y_{62} and G_2.

October 1, 1886.

Time.	h m 8 15	h m 8 30	h m 8 45	h m 9 00	h m 9 15	h m 9 35	h m 10 00	h m 10 30	h m 10 50	h m 11 00	h m 11 15
Y_{62}	16°.19	15°.45	16° 73	17°.24	18°.66	20°.88	23°.25	26°.00	27°.19	27°..71	28°.76
G_1	16.30	18.18	21.32	24.18	26.40	29.05	31.12	33.60	34.24	34.24	34.22
G_2	15.88	16.32	17.38	18 63	20.11	22.88	25.10	27.82	28.87	29.38	29.98
Y_{62}—G_1	—1.11	—2.73	—4.59	—6.94	—7.74	—8.17	—7.87	—7.60	—7.05	—6.53	—5.46
Y_{62}—G_2	—0.69	—0.87	—0.65	—1.39	—1.45	—2.00	—1.85	—1.82	—1.68	—1.67	—1.22

Time.	h m 11 30	h m 12 0	h m 12 30	o m 1 50	h m 2 15	h m 2 50	h m 3 0	h m 3 30	h m 3 45	h m 4 15	
Y_{62}	29°.08	30° 17	30°.62	29°.32	28°.97	28°.72	28°.76	28°.76	28°.98	29°.36	°
G_1	33.76	34.58	31 59	29.72	29 68	29.52	29.93	30.79	31.61	32.11	
G_2	30.44	31.48	31.42	29.85	29.57	29.32	29.46	29.50	29.82	30.12	
Y_{62}—G_1	—4.68	—4.41	—0.97	—0.40	—0.71	—0.80	—1.17	—2.03	—2.63	—2.75	
Y_{62}—G_2	—1.36	—1.31	—0.80	—0.53	—0.60	—0.60	—0.70	—0.74	—0.84	—0.76	

Explanatory Notes.

h m	
8 0	Lighted gas under water tank.
10 41	Turned down gas as low as possible.
11 44	Turned on full flow of gas.
11 57	Turned gas down sufficient to hold the temperature at 30° C.
12 21	Opened door of comparing room.
2 11	Turned on full flow of gas.

EXHIBIT E.

Comparisons of yards R_2 R_3 G_1 and G_5, expressed in divisions of the micrometer of which one division=0.567μ.

Observed data.

From —2° to +10°.

Date.			T.	R_2—R_3	R_2—G_1	R_2—G_5
1886,	Dec.	30.	—2°.44	—403.8 div.	—528.6 div.	—516.7 div.
1887,	Jan.	5.	—1.06	—410.8	—540.3	—516.7
"	"	5.	—0.66	—401.0	—540.3	—526.6
"	"	20.	—0.27	—407.4	—529.4	—514.0
1886,	Dec.	31.	+1.12	—389.1	—503.7	—485.6
1887,	Jan.	30.	+2.31	—364.1	—515.8	—497.0
1886,	Dec.	31.	+4.09	—369.0	—485.9	—474.9
1887,	Jan.	11.	+4.09	—369.8	—473.9	—454.1
"	"	17.	+4.79	—360.3	—465.0	—444.5
"	"	10.	+5.76	—367.8	—487.7	—472.9
"	"	14.	+5.96	—348.5	—469.8	—451.4
"	"	9.	+6.46	—357.8	—472.3	—452.3
"	"	11.	+6 56	—352 3	—488 0	—469.9
"	"	14.	+6.86	—351.2	—469.1	—449.0
"	"	28.	+6.91	—349.0	—461.0	—442.3
"	"	14.	+7.06	—365.9	—486.7	—468.3
"	"	14.	+7.84	—346.9	—457.7	—437.7
"	"	14.	+8.15	—340.9	—447.4	—424.2
"	Jan.	3.	+8.30	—333.8	—439.5	—420 0

From +10° to +20°.

Date.			T.	R_2—R_3	R_2—G_1	R_2—G_5
1887,	Jan.	3.	+11°.26	—341.2	—457.7	—437.5
1886,	Dec.	28.	11.86	—326.3	—414.0	—395.7
1887,	Jan.	3.	13.15	—325.7	—449.0	—430.5
1886,	Dec.	27.	15 12	—294.4	—400.1	—381.9
"	"	27.	15.91	—284.4	—392.7	—376.9
1887,	Apr.	1.	15.91	—289.3	—371.9	—353.3
"	Mar.	27.	17.19	—288.9	—376.4	—359.6
"	"	30.	18.64	—278.9	—365.5	—334.5
"	"	30.	20.32	—270.1	—350.9	—333.2

EXHIBIT E.—*Continued.*

From +20° to +30°.

Date.			T.	R_2—R_8	R_2—G_1	R_2—G_5
1887,	Jan.	16.	+20°.95	—273.0 div.	—353.3 div.	—333.7 div.
"	"	16.	21.45	—265.2	—348.5	—329.2
1886,	Dec.	26.	21.98	—236.4	—314.5	—296.4
1887,	Apr.	8.	22.04	—255.0	—338.2	—317.6
1886,	Dec.	26.	22.66	—240.1	—308.1	—282.9
1887,	Jan.	16.	22.73	—262.7	—357.2	—340.5
1886,	Dec.	22.	23.33	—253.0	—331.0	—312.0
1887,	Mar.	24.	23.53	—254.9	—344.5	—323.4
"	Jan.	12.	23.79	—248.1	—312.1	—301.9
"	"	16.	24.33	—251.7	—336.6	—322.9
"	Mar.	31.	24.93	—232.5	—339.8	—320.3
"	Jan.	26.	25.09	—236.0	—315.4	—295.7
1886,	Dec.	22.	25.83	—235.8	—312.5	—291.0
1887,	Mar.	24.	26.22	—248.8	—337.6	—320.2
"	Jan.	5.	27.08	—234.4	—318.4	—304.0
"	"	18.	28.36	—212.5	—295.5	—275.5
1886,	Dec.	19.	28.51	—224.9	—290.0	—270.0
1887,	Apr.	18.	29.49	—208.6	—270.1	—253.2
"	"	18.	29.49	—209.8	—279.7	—263.1
1886,	Dec.	20.	29.66	—212.0	—281.0	—261.0
1887,	Apr.	18.	29.88	—205.0	—264.4	—250.9
1886,	Dec.	26.	29.94	—203.6	—256.5	—243.9
1887,	Jan.	23.	29.98	—213.7	—276.6	—259.9
1886,	Dec.	26.	30.05	—199.2	—260.0	—245.4

From +30° to 40°.

Date.			T.	R_2—R_3	R_2—G_1	R_2—G_5
1887,	Mar.	29.	30°.29	—223.1 div.	—304.0 div.	—282.7 div.
"	Apr.	6.	30.37	—193.7	—247.9	—227.7
"	"	13.	31.92	—201.9	—254.0	—235.7
"	Jan.	24.	32.13	—202.9	—261.1	—245.6
1886,	Dec.	18.	32.50	—198.0	—257.1	—238.2
1887,	Jan.	13.	32.92	—202.6	—291.5	—273.7
"	Mar	30.	33.01	—189.6	—243.7	—216.5
"	Jan.	2.	33.06	—189.3	—246.1	—223.0
1886,	Dec.	18.	33.12	—196.7	—252.3	—233.3
1887,	Jan.	2.	33.13	—184.2	—258.9	—239.6
"	"	2.	33.88	—188.8	—253.8	—236.8
"	Apr.	14.	34.08	—184.2	—245.4	—224.1

EXHIBIT E.—*Continued.*

From +30° to 40°.

Date.			T. °	R_2—R_3	R_2—G_1	R_2—G_5
1887,	Apr.	21.	34.61	—170.6 div.	—226.0 div.	—210.6 div.
"	"	21.	34.69	—173.0	—228.6	—209.6
"	"	12.	35 00	—181.6	—258.5	—234.5
"	"	21.	35.00	—174.6	—231.7	—212.8
"	Jan.	25.	35.22	—173.6	—220.6	—201.7
"	Apr.	7.	35.58	—172.8	—223.2	—254.4
"	"	7.	35.72	—170.1	—209.7	—189.2
"	Mar.	20.	36.36	—168.6	—219.6	—201.9
"	Apr.	7,	36.61	—170.6	—234.0	—214.5
"	"	27.	36.76	—163.0	—219.4	—200.8
"	Mar.	25.	36.86	—158.5	—227.6	—208.3
"	Apr.	7.	36.96	—157.5	—211.8	—192.1
"	Jan.	7.	37.21	—158.6	—214.4	—193.5
1886,	Dec.	21.	37.90	—158.8	—213.3	—194.4
"	"	21.	38.08	—152 1	—211.7	—193.2
1887,	Jan.	26.	38.28	—155.3	—202.3	—184.9
"	"	26.	38.35	—155.1	—205.9	—188.5
"	Apr.	7.	38.37	—172.1	—208.8	—178.8
"	Jan.	21.	38.52	—151.6	—221.7	—203.4
"	Apr.	4.	38.84	—165.0	—220.0	—203.1
"	Mar.	22.	38.99	—149.5	—219.5	—198.3
"	Apr.	8.	39.59	—162.1	—220.9	—201.8
"	Jan.	26.	39.84	—146.0	—191.1	—172.9

From +40° to +50°.

Date.			T. °	R_2—R_3	R_2—G_1	R_2—G_3
"	Mar	19.	39.95	—145.9	—205.6	—178.1
1886,	Dec.	23.	40.88	—136.6	—195.0	—181.0
"	Dec.	23.	41.19	—129.8	—197.6	—179.0
1887,	Apr.	5.	41.25	—147.9	—196.1	—178.5
"	Apr.	5.	41.65	—141.1	—197.5	—177.7
"	Mar.	17,	41.87	—143.0	—181.5	—162.8
"	Apr.	5.	41 92	—144.6	—197.2	—180.6
1886,	Dec.	24.	42.02	—112.5	—181.0	—163.0
1887,	Mar.	17.	42.37	—128.6	—186.6	—168.5
"	Apr.	7.	42.44	—141.2	—191.0	—171.2
"	Apr.	11.	42.59	—118.2	—183.6	—166.2
1886,	Dec.	24.	42.66	—115.4	—153.4	—135.9
1887,	Mar.	17.	43.42	—130.4	—180.7	—160.4

EXHIBIT E.—*Continued.*

From +40° to +50°.

	Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887,	Mar. 21.	43°.42	—122.1 div.	—158·9 div.	—140.3 div.
		45.72	—133.0	—158.7	—138.7
"	Apr. 19.	46.02	—109.0	—139.2	—110.8
1887,	" 10.	46.82	—100.4	—142.3	—123.1
"	" 17.	47.07	—115.3	—148.3	—131.7
"	Apr. 20.	47.82	—111.5	—151.4	—133.5
"	Mar. 20.	47.82	— 96.7	—126.4	—107.7
"	Apr. 17.	48.07	—106.3	—143.6	—125.1
"	Mar. 20.	48.12	— 95.2	—134.3	—119.0
"	" 20.	48.12	— 94.9	—130.7	—109.7
"	May 13.	48.37	— 90.0	—123.9	—106.9
"	" 13.	48.52	— 94.4	—122.5	—106.7
"	Mar. 20.	48.62	— 95.6	—134.4	—116.1
"	Apr. 11.	48.82	— 90.1	—140.8	—120.9
"	Mar. 17.	48.82	— 83.5	—138.7	—123.4
"	May 16.	49.25	— 83.6	—116.6	—100.3
"	Apr. 29.	49.87	— 87.8	—127.7	—102.5

From +50° to +60°.

	Date.	T.	R_2—R_3	R_2—G_1	R_1—G_5
1887,	Apr. 25.	49°.92	— 82.4	—110.4	— 91.1
"	May 16.	49.92	— 84.6	—121.2	—103.0
"	Apr. 25.	50.02	— 82.5	—112.7	— 93.0
"	May 12.	50.77	— 81.8	—105.7	— 87.3
"	" 12.	50.77	— 81.7	—109.6	— 87.8
"	Apr. 17.	51.12	— 87.9	—117.3	— 99.1
"	May 12.	51.27	— 74.8	—111.0	— 93.4
"	Apr. 29.	51.67	— 84.0	—124.6	—106.5
"	" 29.	51.91	— 77.5	—107.0	— 88.2
"	May 12.	57.92	— 80.6	—114.9	— 96.8
"	" 1.	51.97	— 81.6	—104.8	— 77.0
"	" 1.	51.22	— 74.7	—105.4	— 84.2
"	Apr. 27.	52.71	— 60.4	— 87.4	— 66.0
"	" 29.	52.81	— 70.0	—113.5	— 93.9
"	" 29.	52.81	— 64.9	—100.2	— 82.4
"	" 29.	52.91	— 60.1	—101.8	— 84.6
"	" 27.	52.91	— 62.6	— 86.0	— 66.0
"	" 29.	53.01	— 71.7	—110.7	— 91.9
"	" 29.	53.01	— 69.0	—105.6	— 87.5

EXHIBIT E.—*Continued.*

From +50° to +60°.

Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887, Apr. 29.	53°.16	— 65.1 div.	—108.3 div.	— 89.7 div.
" " 11.	53.49	— 67.2	— 86.3	— 70.4
" " 24.	54 76	— 49.6	— 79.6	— 59.9
" " 24.	54.85	— 50.8	— 77.7	— 59.1
" " 24.	54.87	— 48.5	— 71.7	— 54.9
" " 24.	54.90	— 57.0	— 80.8	— 64.1
" " 24.	54.93	— 52.3	— 80.8	— 59.7
" " 24.	54.95	— 51.2	— 81.3	— 66.3
" " 24.	54.95	— 54.5	— 85.3	— 65.8
" " 24.	55.00	— 46.5	— 71.1	— 52.5
" " 27.	55.00	— 48.7	— 76.1	— 54.1
" " 27.	55.05	— 46.2	— 66.5	— 46.0
" " 27.	55.05	— 50.4	— 70.5	— 52.2
" " 24.	55.10	— 48.4	— 66.6	— 48.5
" May 30.	55.81	— 44.3	— 69.2	— 49.5
" " 30.	55.87	— 44.3	— 71.4	— 49.9
" " 30.	55.92	— 43.3	— 69.8	— 49.8
" " 19.	56.44	— 39.1	— 51.9	— 36.5
" " 30.	56.57	— 42.6	— 73.3	— 56.2
" " 30.	56.81	— 40.2	— 71.7	— 52.3
" " 15.	57.05	— 44.0	— 67.5	— 50.2
" " 8.	57.35	— 40.5	— 56.7	— 36.9
" " 30.	57.51	— 35.0	— 57.8	— 39.7
" " 15.	57.64	— 38.2	— 61.3	— 43.2
" " 1.	57.99	— 32.5	— 53.3	— 34.5
" " 1.	58.04	— 28.1	— 44.2	— 26.4
" " 19.	88.44	— 31.7	— 58.3	— 36.6
" " 15.	58.62	— 27.9	— 35.9	— 16.5
" " 15.	58.72	— 27.7	— 33.9	— 16.7
" " 23.	58.73	— 34.6	— 66.4	— 49.5
" " 19.	58.92	— 31.5	— 49.8	— 32.1

From +60° to +70°.

Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887, May 18.	60°.22	—21.9	—30.6	—10.6
" " 17.	60.27	—12.5	—26.5	— 7.0
" " 18.	60.32	—17.0	—34.7	—17.5
" " 17.	60.47	—18.4	—25.4	— 8.7
" " 18.	60.82	— 9.4	—18.1	+ 0.2
" " 4.	60.92	— 7.7	—18.5	+ 0.8

EXHIBIT E.—*Continued.*

From +60° to +70°.

Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887, May 4.	60°.97	—11.8 div.	—20.4 div.	+ 1.2 div.
" " 10.	62.01	— 7.3	—11.6	+ 9.3
" " 10.	62.16	— 9.2	—11.4	+ 9.3
" " 6.	62.16	+ 0.4	— 5.9	+14.0
" " 10.	62.41	— 3.2	—14.8	+ 9.8
" " 10.	62.46	— 4.1	— 8.4	+ 3.0
" " 6.	62.61	— 4.2	—19.0	— 2.8
" " 9.	62.16	— 5.5	—10.7	+ 5.3
" " 9.	63.31	— 8.7	—10.0	+ 7.6
" " 20.	63.31	+ 4.7	— 7.9	+10.2
" " 9.	63.41	— 0.6	— 6.8	+14.0
" " 9.	63.51	— 3.4	—16.5	+ 3.6
" " 9.	63.66	— 2.6	—15.9	+ 4.6
" " 6.	63.71	— 4.4	—17.7	+ 0.2
	64.05	— 7.3	—10.7	+ 2.6
" " 4.	64.26	+ 1.1	— 1.7	+18.7
" " 22.	66.71	+13.2	+12.2	+32.7
" " 22.	67.01	+17.1	+14.9	+33.7
" " 22.	67.41	+19.7	+18.9	+35.7
" " 22.	67.41	+22.4	+14.6	+34.1
" " 22.	67.51	+25.1	+18.5	+37.5
" " 11.	67.61	+34.6	+26.2	+46.1
" " 21.	67.61	+29.1	+32.8	+53.1
" " 11.	67.76	+29.5	+33.4	+53.0
" " 11.	68.51	+30.8	+24.5	+44.8
" " 24.	68.81	+34.4	+30.6	+50.8
" " 24.	68.81	+33.0	+28.9	+49.6
" " 11.	68.81	+30.5	+38.5	+59.4
" " 11.	68.81	+33.8	+42.0	+63.1
" " 26.	68.86	+38.3	+35.9	+53.3
" " 11.	68.86	+30.3	+35.5	+56.3
" " 11.	68.91	+37.7	+44.9	+63.5
" " 11.	68.96	+31.5	+40.3	+56.5
" " 11.	69.11	+37.2	+38.8	+58.9

EXHIBIT E.—*Continued.*

From +70° to +79°.

	Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887	May 15.	72°.41	+ 56.6 div.	+ 58.8 div.	+ 70.7 div.
"	July 12.	72.57	+ 54.8	+ 53.4	+ 80.0
"	May 25.	72.62	+ 53.6	+ 68.2	+ 86.9
"	July 12.	72.68	+ 53.0	+ 53.0	+ 72.4
"	" 12.	72.81	+ 53.7	+ 52.8	+ 70.7
"	" 11.	73.11	+ 55.1	+ 63.8	+ 83.8
"	" 11.	73.13	+ 63.9	+ 74.9	+ 94.3
"	" 11.	73.15	+ 63.3	+ 75.1	+ 92.8
"	" 11.	73.28	+ 62.8	+ 72.0	+ 92.9
"	" 11.	73.32	+ 62.0	+ 75.1	+ 93.0
"	" 11.	73.40	+ 64.8	+ 75.8	+ 93.9
"	" 11.	73.40	+ 59.3	+ 74.7	+ 95.2
"	" 11.	73.45	+ 60.0	+ 74.9	+ 90.7
"	" 11.	73.49	+ 62.2	+ 73.8	+ 91.9
"	" 11	73.56	+ 61.7	+ 74.1	+ 92.0
"	" 10.	75.95	+ 79.0	+ 98.3	+117.8
"	" 10.	75.97	+ 78.3	+ 99.7	+118.3
"	" 10.	76.01	+ 80.1	+ 99.8	+118.3
"	" 10.	76.05	+ 80.2	+102.9	+120.7
"	" 10.	76.30	+ 82.5	+ 99.4	+120.2
"	" 10.	76.33	+ 75.4	+ 98.2	+117.2
"	" 10	76.57	+ 82.8	+101.1	+120.2
"	" 10.	76.41	+ 79.2	+101.8	+120.4
"	" 10.	76.77	+ 82.8	+103.5	+121.9
"	" 10.	76.77	+ 82.8	+107.3	+125.5
"	" 10.	76.79	+ 83.1	+105.4	+122.0
"	" 10.	76.92	+ 81.3	+108.0	+126.0
"	" 10.	76.92	+ 85.4	+107.5	+126.2
"	" 10.	77.15	+ 90.1	+112.2	+130.2
"	" 10.	77.17	+ 83.4	+111.3	+129.6
"	" 10.	77.23	+ 90.1	+112.7	+130.4
"	" 10.	77.38	+ 89.3	+110.8	+128.0
		77.64	+ 91.4	+111.6	+130.0
		77.83	+ 94.9	+112.6	+134.0
"	July 10.	78.13	+ 88.0	+109.7	+126.9

EXHIBIT E.—*Continued.*

From +79° to +93°.

Date.			T.	R_2—R_3	R_2—G_1	R_1—G_5
1887,	July	8.	79°.66	+102.5 div.	+128.9 div.	+144.0 div
“	“	8.	79.68	+104.6	+130.1	+146.2
“	“	8.	79.72	+102.8	+130.0	+145.0
“	“	7.	79.74	+101.7	+127.3	+145.7
“	“	8.	79.86	+103.5	+130.2	+145.5
“	“	7.	79.86	+101.4	+121.4	+139.3
“	“	7.	80.02	+100.0	+126.7	+141.4
“	“	7.	80.05	+101.8	+127.1	+140.8
“	“	7.	80.26	+102.6	+123.7	+143.3
“	“	4.	82.07	+115.0	+148.5	+168.7
“	“	4.	82.09	+112.7	+141.5	+162.9
“	“	4.	82.12	+116.9	+147.9	+164.9
“	“	4.	82.24	+116.5	+148.4	+164.9
“	“	4.	82.66	+118.3	+147.2	+164.6
“	“	3.	84.11	+129.6	+163.7	+182.0
“	“	3.	84.11	+131.1	+165.1	+181.3
“	“	3.	84.16	+124.0	+159.1	+179.1
“	“	3.	84.19	+127.0	+161.1	+178.0
“	“	3.	84.34	+125.9	+156.6	+174.3
“	“	3.	84.49	+124.8	+159.0	+177.3
“	“	3.	84.57	+126.2	+158.5	+174.4
“	“	3.	84.66	+135.7	+164.5	+182.0
“	“	3.	84.76	+135.1	+155.9	+181.5
“	“	3.	84.78	+128.2	+164.5	+175.7
“	“	3.	84.80	+130.2	+162.5	+180.0
“	“	3.	84.80	+128.8	+158.4	+177.0
“	“	3.	84.86	+128.9	+160.2	+176.3
“	“	3.	84.87	+129.1	+162.3	+179.1
“	“	3.	84.90	+133.8	+162.2	+180.2
“	“	6.	86.29	+145.1	+185.9	+204.4
“	“	6.	86.34	+142.8	+185.0	+198.6
“	“	6.	86.61	+141.9	+185.5	+202.5
“	“	6.	86.62	+141.0	+177.5	+195.3
“	“	6.	86.87	+148.8	+190.4	+207.7
“	“	6.	87.07	+141.8	+181.6	+199.9
“	“	6.	87.16	+142.1	+179.1	+196.3
“	“	6.	87.16	+142.1	+180.5	+199.8
“	“	6.	87.81	+142.4	+181.5	+199.3
“	“	1.	88.46	+150.1	+193.0	+212.5
“	“	1.	88.56	+151.2	+194.0	+213.0
			88.68	+148.4	+197.9	+219.9

EXHIBIT E.—*Continued.*

From +79° to +93°.

Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887, July 1.	88°.71	+151.7 div.	+204.6 div.	+223.4 div.
" " 1.	88.77	+146.3	+193.5	+212.0
" June 30.	88.76	+151.6	+191.2	+209.3
" " 30.	88.76	+154.9	+190.2	+210.5
" " 30.	88.84	+150.9	+191.5	+209.8
" " 30.	88.91	+154.7	+191.6	+209.2
" " 30.	88.91	+154.0	+193.6	+212.0
" July 5.	91.98	+178 8	+226.3	+244.7
" " 5.	92.21	+175.4	+228.3	+246.4
" " 5.	93.29	+182.3	+234.2	+252.9

From —20° to —14°.

Date.	T.	R_2—R_3	R_2—G_1	R_2—G_5
1887, Jan. 4.	—19°.80	—490.0	—645.8	—623.8
" " 9.	—13.80	—457.0	—614.0	—596.3
" " 19.	—11.85	—459.6	—614.0	—588.9

EXHIBIT F.

Results in groups of ten degrees each.

T.	Δ	R$_2$—R$_3$	Δ	Coefficient for 1°.		R$_2$—G$_1$	Δ	Coefficient for 1°.		R$_2$—G$_5$	Δ	Coefficient for 1°.	
4°.58		—366.8 div.				—486.0 div.				—467.8 div.			4.
	11.05		6.87	6.22 div.	3.53μ		9.16	8.29 div.	4.70μ		93.0	8.42 div.	4.77μ
15.53		—298.1				—394.4				—374.8			
	10.52		6.52	6.20	3.52		8.62	8.19	4.64		84.2	8.00	4.54
26.05		—232.9				—308.2				—290.6			
	9.49		5.82	6.13	3.48		7.53	7.94	4.50		77.2	8.14	4.62
35.54		—174.7				—232.9				—213.4			
	9.64		5.99	6.21	3.52		7.34	7.61	4.31		72.8	7.55	4.28
45.18		—114.8				—159.5				—140.6			
	9.34		0.90	6.32	3.58		7.66	8.20	4.65		76.6	8.20	4.65
54.52		— 55.8				— 82.9				— 64.0			
	10.92		6.52	6.26	3.55		8.76	8.41	4.77		87.8	8.43	4.78
64.94		+ 9.4				+ 4.7				+ 23.8			
	10.17		6.39	6.22	3.53		8.49	8.27	4.69		84.3	8.21	4.65
75.21		+ 73.3				+ 89.6				+108.1			
	9.88		5.89	5.96	3.38		7.76	7.85	4.48		76.8	7.77	4.40
85.09		+132.2				+167.2				+184.9			
			Means	6.18 div.	3.51μ			8.09 div.	4.59μ			8 09 div.	4.59μ

EXHIBIT G.

Results in groups of 10 observations each.

		1°=6.17 div.		At 33°.86	Diff. from	1°=8.09 div.		At 33°.86	Diff. from	1°=8.09 div.		At 33°.86	Diff. from
No.	T.	R_2—R_3	Red.	R_2—R_3	mean	R_2—G_2	Red.	R_2—G_2	mean	R_3—G_2	Red.	R_2—G_2	mean
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
1	+ 1.77	—384.3	+198.1	—186 2	—1.5	—507.1	+259.7	—247.4	+1.1	—490.3	+259.7	+230.6	—1.0
2	+ 7.19	349.3	+164.5	184.8	—0.1	464.8	+215.7	249.1	—0.7	445.3	+215.7	229.6	+0.0
3	15.52	298.1	+113.2	184.9	—0.2	394.4	+148.3	246.1	+2.3	374 8	+148.3	226.5	+3.0
4	22.68	254.0	+ 68.9	185.1	—0.4	334.4	+ 90.4	244.0	+4.4	316.0	+ 90.4	225.6	+4.0
5	27.46	223.2	+ 39.5	183.7	+1.0	304.0	+ 51.8	252.2	—3.8	285.4	+ 51.8	233.6	—4.0
6	30.71	204.6	+ 19.4	185.2	—0.6	264.6	+ 25.5	239.1	+9.3	248.0	+ 25.5	222.5	+7.0
7	33.75	186.0	+ 0.7	185.3	—0.6	250.0	+ 0.9	249.1	—0.7	230.0	+ 0.9	228.7	+0.9
8	36.23	166.8	— 14.6	181.3	+3.3	221.2	— 19.2	240.3	+8.1	201.9	— 19.2	221.1	+8.5
9	38.68	156.8	— 29.7	186.5	—1.8	211.5	— 39.0	250.5	—2.1	191.9	— 39.0	230.9	—1.0
10	41.56	137.1	— 47.5	184.6	+0.1	192.9	— 62.3	255.3	—6.9	174.1	— 62.3	236.4	—6.9
11	45.34	115.2	— 70.9	186.1	—1.4	154.3	— 92.9	247.2	+1.2	134.8	— 92.9	227.7	+1.8
12	48.66	92.1	— 91.3	183.4	+1.2	131.3	—119.7	251.0	—2.6	113.1	—119.7	232.8	—3.3
13	50.93	81.7	—105.3	187.0	—2.3	113.5	—138.2	251.7	—3.3	94.6	—138.2	232.8	—3.3
14	52.75	68.0	—116.5	184.5	+0.2	102.4	—152.7	255.2	—6.8	82.3	—152.8	535.1	—5.5
15	+54.77	— 52.6	—129.0	—181.6	+3.0	— 79.1	—169.0	—248.0	+0.5	—60.7	—169.0	—229.7	—0.2

EXHIBIT G.—*Continued.*

		1°=6.17 div.		At 33°.86	Diff. from	1°=8.09 div.		At 33°.86	Diff. from	1°=8.09 div		At 33°.86	Diff. from
No.	T.	R_2—R_3	Red.	R_2—R_3	mean	R_2—G_1	Red.	R_2—G_1	mean	R_2—G_5	Red.	R_2—G_5	mean
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
16	+55.97	—44 3	+110.3	+ 66.0	+1.9	— 67.8	+144.6	+ 76.8	—0.2	— 49.1	+144.6	+ 95.6	+0.2
17	58.20	—32.8	96.5	63.7	—0.3	— 51.8	+126.5	74.7	—2.2	— 33.2	+126.5	93.3	—2.0
18	61.03	—11.5	79.0	67.5	+3.4	— 20.3	+103.6	83.2	+6.2	— 0.9	+103.6	102.7	+7.4
19	63.15	— 3.2	65.9	62.7	—1.4	— 12.8	+ 86.4	73.6	—3.3	+ 5.5	+ 86.4	91.8	—3.5
20	66.73	+18.4	43.9	62.3	—1.7	+ 15.9	+ 57.5	73.4	—3.6	+ 34.7	+ 57.5	92.2	—5.0
21	68.45	+33.7	33.3	67.0	+2.9	+ 36.0	+ 43.7	79.7	+2.8	+ 55.6	+ 43.7	99.3	+4.0
22	72.91	+57.8	+ 5.7	63.5	—0.6	+ 64.7	+ 7.5	72.2	—4.8	+ 83.8	+ 7.5	91.3	—1.0
23	74.76	70.8	— 5.7	65.1	+1.1	+ 87.3	— 7.4	79.9	+3.0	+105.9	— 7.4	98.5	+3.2
24	76.76	82.6	— 18.0	64.6	+0.6	+105.6	— 23.6	82.0	+5 0	+123.9	— 23.6	100.3	+5.0
25	78.68	96.9	— 29.9	67.7	+2.6	+120.4	— 39.2	81.2	+4.3	+137.6	— 39.2	98.4	+3.2
26	81.54	111.5	— 47.5	64.0	—0.1	+139.6	— 62.3	77.3	+0.3	+157.3	— 62.3	95.0	—0.3
27	84.48	128.8	— 65.6	63.2	—0.9	+160.7	— 86.0	74.7	—2.2	+178.3	— 86.0	92.3	—3.0
28	85.92	138.1	— 74.5	63.6	—0.5	+174.9	— 97.7	77.2	+0.2	+192.1	— 97.7	94.4	—0.8
29	88.28	148.1	— 89.0	59.1	—4.9	+190.6	—116.7	73.9	—3.1	+209.6	—116.7	91.4	—3.9
30	+90.69	+166.0	—104.0	+62.0	—2.1	+210.9	—136.3	74.6	—2.4	+229.0	—136.3	+92.7	—2.5

EXHIBIT H.

Equations of Condition for $T=33^{\circ}.86$.

R_2-R_3		R_2-G_1		R_2-G_5	
-1.5 div.	$=\Delta_a+32.10\Delta_b$	$+1.1$ div.	$=\Delta_a+32.10\Delta_b$	-1.0 div.	$=\Delta_a+32.10\Delta_b$
-0.1	$=\Delta_a+26.67\Delta_b$	-0.7	$=\Delta_a+26.67\Delta_b$	$+0.0$	$=\Delta_a+26.67\Delta_b$
-0.2	$=\Delta_a+18.34\Delta_b$	$+2.3$	$=\Delta_a+18.34\Delta_b$	$+3.0$	$=\Delta_a+18.34\Delta_b$
-0.4	$=\Delta_a+11.18\Delta_b$	$+4.4$	$=\Delta_a+11.18\Delta_b$	$+4.0$	$=\Delta_a+11.68\Delta_b$
$+1.0$	$=\Delta_a+6.40\Delta_b$	-3.8	$=\Delta_a+6.40\Delta_b$	-4.0	$=\Delta_a+6.40\Delta_b$
-0.6	$=\Delta_a+3.15\Delta_b$	$+9.3$	$=\Delta_a+3.15_b\Delta$	$+7.0$	$=\Delta_a+3.15\Delta_b$
-0.6	$=\Delta_a+0.11\Delta_b$	-0.7	$=\Delta_a+0.11\Delta_b$	$+0.9$	$=\Delta_a+0.11\Delta_b$
$+3.3$	$=\Delta_a-2.37\Delta_b$	$+8.1$	$=\Delta_a-2.37\Delta_b$	$+8.5$	$=\Delta_a-2.37\Delta_b$
-1.8	$=\Delta_a-4.82\Delta_b$	-2.1	$+\Delta_a-4.82\Delta_b$	-1.0	$=\Delta_a-4.82\Delta_b$
$+0.1$	$=\Delta_a-7.70\Delta_b$	-6.9	$=\Delta_a-7.70\Delta_b$	-6.9	$=\Delta_a-7.70\Delta_b$
-1.4	$=\Delta_a-11.48\Delta_b$	$+1.2$	$=\Delta_a-11.48\Delta_b$	$+1.8$	$=\Delta_a-11.48\Delta_b$
$+1.2$	$=\Delta_a-14.80\Delta_b$	-2.6	$=\Delta_a-14.80\Delta_b$	-3.3	$=\Delta_a-14.80\Delta_b$
-2.3	$=\Delta_a-17.07\Delta_b$	-3.3	$=\Delta_a-17.07\Delta_b$	-3.3	$=\Delta_a-17.07\Delta_b$
$+0.2$	$=\Delta_a-18.89\Delta_b$	-6.8	$=\Delta_a-18.89\Delta_b$	-5.5	$=\Delta_a-18.89\Delta_b$
$+3.0$	$=\Delta_a-20.90\Delta_b$	$+0.5$	$=\Delta_a-20.90\Delta_b$	$-0\ 2$	$=\Delta_a-20.90\Delta_b$

Normal Equations	Normal Equations.	Normal Equations.
$0=1.5\Delta_a-0.08\Delta_b$		
$-83.3=-0.08\Delta_a-3777.3\Delta_b$	$+355.7=3777.3\Delta_b$	$+259.6=1766.6\Delta_b$
$\Delta_b=-0.022\Delta_a$	$\Delta_b=+0.094\Delta_b$	$\Delta_b=+0.088\Delta_b$
$b=6.19$ div.	$b=8.00$ div.	$b=8.00$ div.

Equations of Condition for $T=73^{\circ}.84$.

R_2-R_3		R_2-G_1		R_2-G_5	
$+1.9$ div.	$=\Delta_a+17.87\Delta_b$	$-0\ 2$ div.	$=\Delta_a+17\ 87\Delta_b$	$+0.2$ div	$=\Delta_a+17.87\Delta_b$
-0.3	$=\Delta_a+15.64\Delta_b$	-2.2	$=\Delta_a+1.645\Delta_b$	-2.0	$=\Delta_a+15.64\Delta_b$
$+3.4$	$=\Delta_a+12.81\Delta_b$	$+6.2$	$=\Delta_a+12.81\Delta_b$	$+7.4$	$=\Delta_a+12.81\Delta_b$
-1.4	$=\Delta_a+10.69\Delta_b$	-3.3	$=\Delta_a+10.69\Delta_b$	-3.5	$=\Delta_a+10.69\Delta_b$
-1.7	$=\Delta_a+7.11\Delta_b$	-3.6	$=\Delta_a+7.11\Delta_b$	-3.0	$=\Delta_a+7.11\Delta_b$
$+2.9$	$=\Delta_a+5.40\Delta_b$	$+2.8$	$=\Delta_a+5.40\Delta_b$	$+4.0$	$=\Delta_a+5.40\Delta_b$
-0.6	$=\Delta_a+0.93\Delta_b$	-4.8	$=\Delta_a+0.93\Delta_b$	-4.0	$=\Delta_a+0.93\Delta_b$
$+1.1$	$=\Delta_a-0.92\Delta_b$	$+3.0$	$=\Delta_a-0.92\Delta_b$	$+3.2$	$=\Delta_a+0.92\Delta_b$
$+0.6$	$=\Delta_a-2.92\Delta_b$	$+5.0$	$=\Delta_a-2.92\Delta_b$	$+5.0$	$=\Delta_a-2.92\Delta_b$
$+2.6$	$=\Delta_a-4.84\Delta_b$	$+4.3$	$=\Delta_a-4.84\Delta_b$	$+3.2$	$=\Delta_a-4.84\Delta_b$
-1.0	$=\Delta_a-7.70\Delta_b$	$+0.3$	$=\Delta_a-7.70\Delta_b$	-0.3	$=\Delta_a-7.70\Delta_b$
-0.9	$=\Delta_a-10.64\Delta_b$	-2.2	$=\Delta_a-10.64\Delta_b$	-3.0	$=\Delta_a-10.64\Delta_b$
-0.5	$=\Delta_a-12.08\Delta_b$	$+0.2$	$=\Delta_a-12.08\Delta_b$	-0.8	$=\Delta_a-12.08\Delta_b$
$-4\ 9$	$=\Delta_a-14.44\Delta_a$	-3.1	$=\Delta_a-14.44\Delta_b$	-3.9	$=\Delta_a-14.44\Delta_b$
-2.1	$=\Delta_a-16.85\Delta_b$	-2.4	$=\Delta_a-16.85\Delta_b$	-2.5	$=\Delta_a-16.85\Delta_b$

$3356.0=3533.2\Delta_b$	$+57.=1766.6\Delta_b$	
$\Delta_b=+0.095$	$\Delta_b=+0.032_b$	$+135.5=1766\ 6_b$
$b=6.07$ div.	$b=8.06$ div.	$\Delta_b=+0.076$
		$b=8.01$ div.

EXHIBIT I.

Relations between R_2 R_3 and G. in groups of 10 observations each, reduced to 62°.0 Fahr.

62°—T.	R_2—R_3	At 62°.0 R_2—R_3	Mean of groups.
+60°.23	—384.3 div.	—12.7 div.	
54.81	349.3	11.1	
46.48	298.1	11.3	
39.32	254.0	11.4	
34.54	223.2	10.1	--11.32 div.
31.29	204.6	11.5	
28.25	186.0	11.7	
25.77	166.8	7.8	
23.32	156.8	12.9	
20.44	137.1	11.0	—10.98
16.66	115.2	12.4	
13.34	92.1	9.8	
11.07	81.7	13.4	
9.25	68.0	10.9	
7.23	52.6	8.0	—10.90
6.03	44.3	7.1	
3.80	32.8	9.4	
+ 0.97	11.5	5.5	
— 1.15	— 3.2	10.3	
4.73	+ 18.4	10.8	— 8.62
6.45	33.7	1.1	
10.91	57.8	9.5	
12.76	70.8	7.9	
14.76	82.6	8.5	
16.68	96.9	6.0	
19.54	111.5	9.0	
22.48	128.8	9.9	
23.92	138.1	9.5	
26.28	148.1	14.1	
—28.69	+166.0	—11.1	—10.72

EXHIBIT I.—*Continued.*

Relations between R_2 and G_1.

62°—T.	R_2—G_1	At 62°.0 R_2—G_1	Mean of groups.
+60°.23	—507.1 div.	—19.8 div.	
54.81	464.8	21.4	
46.48	394.4	18.4	
39.32	334.4	16.3	
34.54	304.0	24.5	—20.08 div.
31.29	264.6	11.5	
28.25	250.0	21.5	
25.77	221.2	12.7	
23.32	211.5	22.8	
20.44	192.9	27.5	—19.20
16.66	154.3	19.5	
13.34	131.3	23.4	
11.07	113.5	23.9	
9.25	102.4	27.6	
7.23	179.1	20.6	—23.00
6.03	67.8	19.0	
3.80	51.8	21.1	
+ 0.97	20.3	12.5	
— 1.15	— 12.8	22.1	
4.73	+ 15.9	22.4	—19.42
6.45	36.0	16.2	
20.91	64.7	23.6	
12.76	87.3	15.9	
14.76	105.6	13.8	
16.68	120.4	14.5	—16.80
19.54	139.6	18.5	
22.48	160.7	21.2	
23.91	174.9	18.6	
26.28	190.6	22.0	
—28.69	+210.9	—21.2	—20.30

EXHIBIT I.—*Continued.*

Relations between R_2 and G_5.

62°—T	R_2—G_5	At 62°.0 R_2—G_5	Mean of groups.
+60°.23	—490.3 div.	—3.0 div.	
54.8r	445.3	—1.9	
46.48	374.8	+1.2	
39.32	316.0	+2.1	
34.54	284.5	—5.9	—1.50 div.
32.29	248.0	+5.1	
28.25	230.0	—1.5	
25.77	201.9	+6.6	
23.32	191.9	—3.2	
20.44	174.1	—8.7	—0.34
16.66	134.8	+0.0	
13.34	113.1	—5.2	
11.07	94.6	—5.0	
9.25	82.3	—7.5	
7.23	60.7	—2.2	—3.98
6.03	49.1	—0.3	
3.80	33.2	—2.5	
+ 0.97	— 0.9	+6.9	
— 1.15	+ 5.5	—3.8	
4.73	34.7	—3.6	—0.66
6.45	55.6	+3.4	
10.91	83.8	—4.5	
12.76	105.9	+2.7	
14.76	123.9	+4.5	
16.68	137.6	+2.7	+1.76
19.54	157.3	—0.8	
22.48	178.3	—3.6	
23.92	192.1	—1.4	
26.28	209.6	—3.0	
28.69	229.0	—3.1	—2.38

EXHIBIT J.

Equations between R_2, R_3 and G at $62.\breve{0}$ Fahr.

$$\begin{aligned}
-11.32 \text{ div.} &= a+47.08\Delta_b \\
-10.98 &= a+25.81\Delta_b \\
-10.90 &= a+11.51\Delta_b \\
-8.62 &= a+0.98\Delta_b \\
-7.60 &= a-12.31\Delta_b \\
-10.72 &= a-24.18\Delta_b
\end{aligned}$$

Normal Equations.

$$\begin{aligned}
-60.14 \text{ div.} &= 6a+48.89\Delta_b \\
-597.47 &= 48.89a+3764.46\Delta_b \\
-10.02 &= a+8.15\Delta_b \\
-12.22 &= a+77.00\Delta_b \\
-2.20 &= +68.85\Delta \\
b &= -0.033 \text{ div.} \\
a &= -9.78 \text{ div.}
\end{aligned}$$

$$\begin{aligned}
-20.08 \text{ div.} &= a+47.08\Delta_b \\
-19.20 &= a+25.81\Delta_b \\
-23.00 &= a+11.51\Delta_b \\
-19.62 &= a+0.98\Delta_b \\
-16.80 &= a-12.31\Delta_b \\
-20.30 &= 2-24.18\Delta_b
\end{aligned}$$

Normal Equations.

$$\begin{aligned}
-19.83 \text{ div.} &= a+8.15\Delta_b \\
-21.01 &= a+77.00\Delta_b \\
-1.18 &= 68.85\Delta_b \\
\Delta b &= -0.17 \text{ div.} \\
a &= -19.70 \text{ div.}
\end{aligned}$$

$R_2 - G_5$

$$\begin{aligned}
-1.50 \text{ div.} &= a+47.08\Delta_b \\
-0.34 &= a+25.81 \\
-3.98 &= a+11.51 \\
-0.66 &= a+0.98 \\
+1.76 &= a-12.31 \\
-2.38 &= a-24.18
\end{aligned}$$

Normal Equations.

$$\begin{aligned}
-1.18 \text{ div.} &= a+8.15\Delta_b \\
-2.14 &= a+77.00\Delta_b \\
-0.94 &= 68.85\Delta_b \\
\Delta b &= -0.14 \text{ div.} \\
a &= -1.17 \text{ div.}
\end{aligned}$$

EXHIBIT K.

Continuation of the comparison between yards R_2, R_3 and G_1.

Thermometer No. 1 rests horizontally upon the upper surface of bar R_2. No. 2 rests upon G. Thermometer No. 3 rests upon the bottom of a slot 22 inches in length, cut in a bronze bar which has the same dimensions as R_2. The depth of the slot is about 0.55 inch. The bulb of the thermometer is completely covered with mercury. The opening in the bar is covered with a thin sheet of rubber, and the pressure of another bar, of the same dimensions upon it excludes the outside air except at the instant of reading. Thermometer No. 4 rests upon the edges of a paper box at the same level as the upper surface of R_2 and about two inches above the bed of the comparator. Thermometer No. 5 is supported in a similar manner at the same level as the upper surface of G.

												At 62°.0	
Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_3—G_1	R_2—R_3 plus 9.8 div.	R_2—G_1 plus 1.1 div.
Sept.	h m												
25	6 0	55.9	.....	.....	.....	.....	— 1.7 div.	—51.4 div.	— 53.1 div.	—49.7 div.	— 51.4 div.	— 2.1 div.	— 0.8 div.
25	6 30	55.93	.....	.....	.....	.....	+44.6	— 5.4	— 10.5	—50.0	— 55.1	— 2.6	— 4.8
26	6 10	51.7	.....	.....	.....	.....	+64.8	— 8.6	— 16.0	—73.4	— 80.8	+ 4.0	+ 8.7
26	6 30	51.07	.....	.....	.....	.....	+63.6	— 6.8	— 19.6	—70.4	— 83.2	+ 6.4	+ 5.5
26	12 10	54.1	.....	.....	.....	.....	+52.0	— 4.1	— 13.6	—56.1	— 65.4	+ 2.6	— 0.2
27	6 30	53.2	.....	.....	.....	.....	+54.4	—10.3	— 7.9	—64.7	— 62.3	— 0.3	+10.2
27	6 50	53.2	.....	.....	.....	.....	+55.9	—13.2	— 10.9	—69.1	— 66.8	— 4.7	+ 5.7
28	6 0	55.25	.....	.....	.....	.....	+ 3.1	—48.1	— 46.0	—51.2	— 49.1	+ 0.4	+ 6.7 *

EXHIBIT K.

Continuation of the comparisons between yards R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1	At 62°.0 R_2—R_3	At 62°.0 R_1—G_1
Sept.	h m	°										plus 9.8 div.	plus 1.1 div.
28	6 30	55.25	.....	.....	.....	.....	+ 5.1 div.	—48.1 div.	— 46.0 div.	—53.2 div.	— 51.1 div.	— 1.6 div.	+ 4.7 div.*
28	7 00	55.25	.....	.....	.....	.....	+ 4.1	—48.3	— 50.1	—52.4	— 54.2	— 0.8	+ 1.6 *
28	10 00	55.9	56°.0	.....	.....	.....	— 2.6	—52.6	— 55.1	—50.0	— 52.5	— 2.4	— 2.7 *
28	11 00	56.15	56.45	.....	.....	.....	— 0.9	—51.9	— 59.2	—51.0	— 58.3	— 4.9	—12.2
28	11 30	56.35	56.9	.....	.....	.....	— 3.4	—51.9	— 57.0	—48.5	— 53.6	— 3.7	—11.1
28	12 00	57.05	57.4	.....	....	.....	— 5.6	—54.1	— 61.2	—48.5	— 55.6	— 8.0	—17.2
28	12 30	57.4	57.9	57°.9	57°.9	.....	— 8.8	—53.8	— 58.0	—45.0	— 49.2	— 6.7	—14.8
28	1 30	58.0	58.1	58.2	58.2	.....	—11.8	—51.8	— 58.0	—40.0	— 46.0	— 5.4	—13.5
28	2 00	58.0	58.2	58.2	58.2	.....	—16.3	—54.2	— 60.4	—37.9	— 44.1	— 3.3	—12.2
28	2 30	58.4	58.85	58.4	58.2	.....	—14.9	—51.6	— 57.7	—36.7	— 42.8	— 4.6	—16.2
28	4 00	58.7	59.15	9.2	59.3	.....	—17.9	—47.4	— 54.0	—29.5	— 36.1	+ 0.8	—11.5 *
28	4 30	58.7	59 0	59.1	59.1	.....	—17.9	—48.1	— 55.1	—30.2	— 37.2	+ 0.1	—11.8 *
28	5 00	59.0	59.1	59.0	59.0	.....	—21.1	—52.2	— 53.0	—31.1	— 31.9	— 2.7	— 7.3 *
29	6 20	54.4	54.8	54.2	54.3	.....	— 2.3	- 54.7	— 54.1	—52.4	— 51.8	+ 4.5	+ 7.7 *
29	6 33	54.5	54.6	54.2	54.3	.....	+ 0.3	—53.2	— 51.2	—53.5	— 51.5	+ 2.8	+ 9.6 *
26	6 49	54.6	54.5	54.1	54.2	.....	+ 0.4	—53.1	— 52.0	—53.5	— 52.4	+ 2.2	+ 9.5 *
29	7 03	54.8	54.4	54.1	54.1	.....	— 2.0	—51.3	— 51.6	—49.3	— 49.6	+ 5.1	+13.1 *
29	7 18	54.8	54.4	54.0	54.0	.....	+ 0.2	—53.1	— 53.1	—53.3	— 53 3	+1 .1 *	+ 9.4 *
29	8 30	54.67	54.15	73.9	53.9	.....	+ 4.3	—54.0	— 53.1	—58.3	— 57.4	— 2.9 *	+ 7.4 *
29	8 40	54.4	54.2	53.9	54.0	.....	+ 2.8	—55.8	— 52.0	—58.6	— 54.8	— 1.7 *	+ 9.4 *

EXHIBIT K.

Continuation of the comparisons between yards R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_3—G_1	At 62.0° R_2—R_3 □	R_2—G_1
Sept.	h m	°	°	°	°							plus 9.8 div.	plus 1.1 div.
29	8 50	54.15	54.15	53.8	54.2	.....	+ 0.4 div.	—54.5 div.	— 51.9 div.	—54.9 div.	— 55.3 div.	— 3.6 div.*	+ 9.5 div.*
29	9 00	54.3	54.55	53.85	53.8	.....	+ 0.1	—58.7	— 51.8	—58.8	— 51.9	— 1.3 *	+ 9.6 *
29	10 00	54.1	54.0	53.85	53.85	.....	+ 1.9	—56.6	— 56.3	—58.8	— 58.2	— 0.1 *	+ 7.8 *
29	10 30	54.2	54.15	54.0	54.0	.....	+ 1.7	—57.4	— 57.1	—59.1	— 58.8	— 0.9 *	— 6.0 *
29	10 45	54.5	54.4	54.1	54.1	.....	+ 3.3	—58.5	— 56.8	—61.8	— 60.1	— 5.5 *	+ 2.6 *
29	11 00	54.8	54.8	54.9	54.7	.....	+ 2.9	—54.8	— 57.9	—57.7	— 60.8	— 3.3 *	— 1.3 *
29	11 30	54.8	54.9	54.9	54.85	.....	— 1.2	—58.2	— 60.6	—57.0	— 59.4	— 2.6 *	— 0.7 *
29	12 00	55.0	55.1	55.2	55.1	.....	+ 0.6	—56.9	— 61.5	—57.5	— 62.6	— 4.3 *	— 5.1 *
29	12 20	55.4	55.85	56.1	56.1	.....	+ 1.0	—57.1	— 61.6	—58.5	— 64.9	— 7.4	—11.6 *
29	12 45	55.75	56.1	56.65	56.5	.....	+ 1.0	—58.5	— 63.9	—59.5	— 64.0	—11.0	—16.0 *
29	12 55	Placed hands on R_2, R_3 and G for one minute.											
29	12 59	56.3	56.5	57.0	57.0	.....	—10.6	—63.2	— 74.7	—52.6	— 64.1	— 7.5	—18.4
29	1 15	56.8	56.8	57.2	57.3	.....	— 7.7	—66.8	— 68.5	—59.1	— 60.8	—17.1	—17.5
29	1 55	57.2	57.8	58.0	58.0	.....	— 6.8	—58.2	— 65.0	—51.4	— 58.2	—11.8	—23.0
29	2 30	57.8	58.7	59.4	59.2	.....	—11.9	—60.8	— 60.7	—48.9	— 58.8	—13.1	—30.9
29	3 10	58.6	59.9	60.9	60.7	.....	—14.8	—58.4	— 72.4	—43.6	— 57.6	—12.7	—39.5
29	3 40	59.4	60.8	62.8	61.7	.....	—18.9	—58.5	— 76.9	—39.6	— 58.0	—13.7	—47.2
29	6 00	Built fire in comparing room and in the adjoining room. Replenished fires at 9h P. M. No. =81°. No. 3=85°.											
30	6 45	75.9	75.9	71.2	72.1	.....	+ 2.5	+83.4	+143.1	+80.9	+140.6	+ 4.5	+29.0
30	7 00	74.5	73.8	71.35	71.8	.....	+ 5.9	+87.0	+143.0	+81.1	+137.1	+13.4	+44.1

EXHIBIT K.

Continuation of the comparisons between yards R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_3—G_1	At 62°.0 R_2—R_3	 R_2—G_1
Sept.	h m											plus 9.8 div.	plus 1.1 div.
30	9 00	70°.2	70°.2	71°.6	72°.5	.....	+ 26.8 div.	+87.0 div.	+128.5 div.	+60.2 div.	+101.7 div.	+19.2 div.	+36.3 div.
30	10 00	71.1	70.9	69.7	69.9	.....	+ 34.8	+80.5	+121.7	+45.7	+ 86.9	— 0.9	+14.2
30	11 30	70.65	70.15	69.75	69.85	.....	+ 36.5	+84.0	+120.1	+47.5	+ 83.6	+ 3.7	+18.6
30	12 15	70.45	70.15	69.7	69.7	.....	+ 45.3	+86.8	+123.6	+41.5	+ 78.3	— 1.1 *	+13.3 *
30	1 40	70.0	70.1	69.6	69.6	.....	+ 47.4	+84.3	+112.9	+36.9	+ 65.5	— 2.9 *	+ 0.9 *
30	3 00	69.6	69.6	69.0	69.05	.....	+ 49.6	+86.3	+113.1	+36.7	+ 63.5	— 0.6 *	+ 3 0 *
30	4 00	69.55	69.15	68.9	68.9	.....	+ 49.6	+83.7	+ 99.1	+38.1	+ 49.5	+ 1.1 *	— 7.4 *
Oct.													
2	6 50	59.6	59.65	59.2	59.3	.....	+110.7	+84.3	+ 89.6	—26.4	— 21.1	— 1.7 *	— 1.0 *
2	7 10	59.7	59 7	59.3	59.5	.....	— 4.7	—35.5	— 22.0	—30.8	— 17.3	— 6.7 *	+ 2.4 *
2	7 30	59.9	59.85	59.6	59.8	.....	— 5.4	—35.0	— 26 9	—29 6	— 21.5	— 6.2 *	— 3.0 *
2	7 40	Placed both hands on G for one minute—7^h 41^m. Placed both hands on R_1 for one minute.											
2	7 45	59.6	59.85	59.7	59.7	.....	— 14.2	—36.7	— 37.2	—22.5	— 23.0	+ 0.3 *	— 4.5 *
2	7 58	60.0	60.0	60.05	59.9	.....	— 13.4	—34.8	— 33.9	—21.4	— 20.5	+ 0.8 *	— 3.2 *
2	8 45	59.95	59.9	59.8	59.6	.....	— 13.4	—35.5	— 29.9	—22.1	— 16.5	+ 0.4 *	+ 1.6 *
2	12 00	60.8	60.9	60.9	60.9	.....	— 10.0	—33.6	— 29.4	—23.6	— 19.4	— 6.4	— 9.4
2	1 30	61.0	61.1	61.2	61.3	.....	— 10.9	—35.6	— 28.7	—24.7	— 17.8	— 8.7	— 9.4
2	4 00	61.8	71.85	61.9	61.9	.....	— 16.9	—34.7	— 25.2	—17.8	— 8.3	— 5.8	— 6.0

EXHIBIT K.

Continuation of the comparisons between yards R_2, R_3 and G_1.

Placed hands upon R_2 for 1 minute at intervals of 5 minutes.
Comparions of R_2 with stops.

h m	R_2	h m	R_2	h m	R_2	h m	R_2	h m	R_2	h m	R_2
4 46	—0.4 div.	4 51	—10.9 div.	4 56	—21.5 div.	5 1	—31.7 div.	5 6	—36.0 div.	5 11	—40.9 div.
47	—0.2	52	— 9.1	57	—12.1	2	—28.7	7	—35 5		—40.4
48	—1.6	53	— 9.8	58	—19 6	3	—28 6	8	—33.7		—39.5
49	+0.6	54	—11 8	56	—20.3	4	—28.0	9	—33.0		—39.5
Means	—0 5		—10 4		—20 6		—29.3		—34·6		—40.2

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	$R_2—R_3$	$R_2—G_1$	At 62°.0 $R_2—R_2$ plus 9.8 div.	$R_2—G_1$ plus 1.1 div.
Oct.	h m												
3	6 20	60°.9	60°.8	60°.7	60.7	.....	— 1.8 div.	—15.9 div.	— 7.2 div.	—14.1 div.	— 5.4 div.	— 2.5 div.*	+ 5.4 div.*
3	6 40	60.9	60.8	60.4	60.6	.....	+ 0.5	—15.5	— 7.3	—16.0	— 7.8	+ 0.6 *	+ 3 0 *
3	6 50	60.9	60 8	60.5	60.5	.....	+ 0.2	—16 8	— 5.5	—17.0	— 5.7	— 0 4 *	+ 5.1 *
3	7 40	60.95	60.9	60.8	60.7	.....	+ 0.8	—16 9	— 5.9	—17.7	— 6 7	— 1.4 *	+ 3 3 *
3	9 00	61.2	61.15	61 2	61.1	.....	+ 0.4	—17.1	— 9.5	—16.7	— 9,1	— 1.9 *	-- 1.1 *
3	10 00	61.8	62.1	62.15	62.0	.....	— 2.3	—18 2	— 9.8	—15.9	— 7.5	— 4.5	— 7.2 *
3	10 30	62.1	62.3	62.8	62.6	.....	— 2 1	—17.8	— 13.6	—15 7	— 11.5	— 6.5	—12.9
3	11 0	62.4	62.8	63.0	62.85	.....	— 3.9	—17.4	— 11.6	—13.5	— 17.7	— 6.2	—13.0
3	12 0	62.8	63.05	63.3	63.1	.....	— 6.8	—18.8	— 10.7	—12.2	— 3.9	— 2.6	—11.4 *
4	6 30	62.95	62.9	63.0	62.9	62.83	—13.7	—17.9	— 2.7	— 4.2	+ 11.0	— 0.3	+ 4.8 *
4	6 40	62.95	62.95	62.9	62.9	62.83	—11.2	—17.3	— 4.5	— 6.1	+ 6.7	— 2.2	+ 0.1 *
4	7 00	63.0	63.0	62.9	62.9	62.83	— 8.9	—17.1	— 4.0	— 8.2	+ 4.9	— 4.4	— 2.1 *

EXHIBIT K.

Continuation of the comparisons between yards R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1	At 62°.0 R_2—R_3	At 62°.0 R_2—G^1
Oct.	h m	°	°	°	°	°						plus 9.8 div,	plus 1.1 div.
4	9 00	63.25	63.25	63.4	63.25	63.01	— 11.4 div.	—18.1 div.	— 7.1 div.	— 6.7 div.	+ 4.3 div.	— 4.1 div.	— 10.2 div.
4	10 00	64.0	64.0	64.1	64.1	63.36	— 14.7	+19.5	— 9.2	— 4.8	+ 5.5	— 7.3	— 9.6
4	10 30	63.95	64.35	64.6	64.6	64.04	— 17.6	—20.8	— 6.5	— 3.2	+ 11.1	— 5.5	— 6.8
4	11 30	64.15	64.7	65.0	94.95	64.22	— 14.8	—20.1	— 5.8	— 5.3	+ 9.0	— 8.8	— 11.8
4	12 30	64.6	64.85	65.0	65.0	64.49	— 18.5	—21.5	— 9.7	— 3.0	+ 8.8	— 9.3	— 13.2
4	2 20	54.85	65.05	65.15	65.2	64.78	— 23.0	—20.2	— 8.1	+ 2.8	+ 14.9	— 5.1	— 8.7
4	3 50	65.0	65.2	65.1	65.35	64.99	— 23.9	—15.2	— 3.0	+ 8.7	+ 20.9	— 0.1	— 3.9 *
4	4 20	65.1	65.15	65.15	65.3	65.12	— 24.9	—17.5	— 2.7	+ 7.4	+ 22.2	— 2.0	— 2.2 *
4	5 00	65.1	65.15	65.2	65.0	65.16	— 23.4	—14.4	— 0.4	+ 9.0	+ 23.0	— 0.4	— 1.4 *
5	1 45 P.M	62.7	62.6	62.5	62.7	62.46	— 10.6	—18.3	— 4.9	— 7.7	+ 5.7	— 2.3	— 2.0 *
5	2 00	62.7	62.7	62.65	62.75	62.50	— 9.6	—18.9	— 9.2	— 9.3	+ 0.4	— 3.8	— 4.2 *
5	2 20	62.7	62.8	62.85	62.85	62.53	— 9.6	—16.9	— 8.0	— 7.3	+ 1.6	— 1.8	— 3.7 *
5	2 40	62.65	62.8	62.7	62.85	62.55	— 12.9	—19.8	— 10.7	— 6.9	+ 2.2	— 1.1	— 3.1 *
5	3 00	62.65	62.8	62.8	62.8	61.55	— 11.3	—19.3	— 7.2	— 8.0	+ 4.1	— 2.2	— 1.2 *
5	3 15	62.8	62.9	62.7	62.9	62.60	— 12.1	—20.0	— 6.9	— 7.9	+ 5.2	— 3.1	— 1.0 *
5	3 30	62.8	62.8	62.6	62.7	62.62	— 15.1	—19.9	— 8.5	— 4.8	+ 6.6	+ 0.0	+ 1.3 *
5	3 45	62.8	62.8	62.7	62.8	62.64	— 15.5	—20.1	— 8.4	— 4.6	+ 7.2	+ 0.2	+ 1.8 *
5	3 52	Placed one hand on G. and the other on R_2 and R_2 for one minute.											
5	3 35	63.0	62.7	.....	.....	.....	— 17.2	—23.5	— 12.3	— 6.3	+ 4.9	— 2.7	+ 0.3
5	4 00	63.0	62.8	.....	.....	.....	— 18.8	—25.0	— 14.8	— 6.2	+ 4.8	— 2.6	— 0.5
5	4 05	62.8	62.9	.....	.....	.....	— 18.0	—24.5	— 14.0	— 6.5	+ 4.0	— 1.7	— 2.2

EXHIBIT K.

Continuation of the comparisons between yards R_2, R_3 and G_1.

Date Oct.	Time h m	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1	At 62°.0 R_2—R_3 plus 9.8 div.	At 62°.0 R_2—G_1 plus 1,1 div.
5	4 12	62°.8	62°.9	.....	.....	.....	— 16.0 div.	—22.0 div.	— 10.4 div.	— 6.0 div.	+ 5.6 div.	— 1.2 div.	— 0.6 div.
5	4 20	62.8	62.9	.....	.....	.....	— 16.5	—23.5	— 9.0	— 7.0	+ 7.5	— 2.2	+ 1.3
5	3 30	62.8	62.9	.....	.....	.....	— 16.4	—23.0	— 12.0	— 6.6	+ 4.4	— 1.8	— 1.8
5	4 32	....	....	.....	.....	.....	......	......	....	.....	....	.....	......
Placed hands on R_2 for 1 minute at intervals of 5 minutes.													
5	4 33	....	....	.....	.....	.....	— 21.7	......	....	.....	....	.....	......
5	4 38	....	....	.....	.....	.....	— 28.7	......	....	.....	....	.....	......
5	4 43	....	....	.....	.....	.....	— 31.3	......	....	.....	....	.....	......
5	4 48	....	....	.....	.....	.....	— 35.0	......	....	.....	....	.....	......
5	4 53	....	....	.....	.....	.....	— 37.7	......	....	.....	....	.....	......
5	4 58	....	....	.....	.....	.....	— 41.9	......	....	.....	....	.....	.....
6	7 10	59.2	59.1	59°.1	58°.8	59°.43	+ 8.0	—19.3	— 8.0	—27.3	— 16.0	— 0.1	+ 8.5 *
6	7 20	59.2	59.1	58.9	58.9	59.43	+ 6.6	—19.7	— 10.2	—26.3	— 16.8	+ 0.9	+ 7.8 *
6	8 10	59.5	59 5	59.0	59.0	59.59	+ 7.0	—18.6	— 13.2	—25.6	— 20.2	— 0.3	+ 1.2 *
6	8 30	60.1	60.15	60.15	60.15	59.81	+ 7.0	—18.5	— 14.5	—25.5	— 21.5	— 3.9	— 5.4 *
6	9 00	60.7	60.9	61.0	61.1	59.81	+ 7.2	—19.4	— 16.1	—26.6	— 23.3	— 8.7	—13.3 *
6	10 30	61.0	61.5	61.8	62.0	60.62	+ 3.0	—17.8	— 21.5	—20.8	— 24.5	— 4.8	—19.4 *
6	10 45	61.6	61.95	62.25	62.3	60.87	+ 2.2	—18.0	— 21.6	—20.2	— 23.8	— 7.9	—22.3
6	12 0	61.75	62.0	62.5	62.7	61.47	— 0 8	—20.9	— 19.8	—20.1	— 19.3	— 8.7	—18.2
6	4 15	62.8	62.85	63.15	63.0	62.64	— 9.9	—17.8	— 21.4	— 7.9	— 11.5	— 3.1	—17.3
6	4 25	Opened both windows. Temperature of outside air =56°											
6	4 40	62.5	62.7	62.4	62.8	62.60	— 9.5	—18.2	— 9.6	— 8.7	+ 0.8	— 2.0	— 3.8
6	5 10	62.0	62.0	61.5	61.9	62.24	— 9.1	—17.6	— 1.8	— 8.5	+ 7.3	+ 1.3	+ 8.4
6	8 0	60.9	60.5	59.2	59.7	60.17	— 1.9	—18.8	— 6.1	—16.9	+ 4.2	— 0.3	+ 9.1

EXHIBIT L.

Comparison between metres R_2 R_3 and G_1.

												At 62°.0	
Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1	R_2—R_3	R_2—G_1
Oct.	h m											plus 48 div.	minus 140 div.
10	6 30	59°.15	59°.2	59°.3	59°.3	59.23	+ 3.7 div.	—17.1 div.	— 5.7 div.	—20.8 div.	— 8.4 div.	+ 3.3 div.	+ 2.4 div.
10	9 00	59.25	59.35	59.3	59.5	59.19	+ 4.0	—17.2	— 5.0	—21.2	— 9.0	+ 2.2	+ 0.5
11	6 20	61.75	61.45	61.0	61.0	61.50	— 13.3	—18.6	— 6.0	— 5.3	+ 7.3	+ 1.2	— 1.8
11	6 40	61.5	61.4	60.5	60.6	61.52	— 10.3	—19.1	— 5.3	— 8.8	+ 5.0	— 0.6	— 3.7
11	9 30	61.75	61.9	62.1	62.1	61.56	— 11.1	—17.3	— 5.2	— 6.2	+ 5.9	+ 0.3	— 7.2
11	11 45	61.8	61.65	61.6	61.6	61.70	— 10.9	—19.1	— 4.4	— 8.2	+ 6.5	— 2.0	— 4.5
11	3 20	61.15	61.0	60.8	60.9	61.23	+ 1.8	— 6.1	+ 3.7	— 7.9	+ 1.9	+ 2.7	— 3.2
11	3 40	61.1	61.0	61.0	61.0	61.16	— 2.7	— 9.0	+ 4.4	— 6.3	+ 7.1	+ 4.6	+ 2.0
11	4 00	61.0	60.9	60.7	60.7	61.00	+ 0.1	—10.0	+ 4.0	—10.1	+ 4.1	+ 1.5	— 0.2
12	6 25	53.7	53.2	53.0	53.1	53.42	+ 61.5	+ 6.3	+ 9.1	—55.2	— 52.4	+ 5.9	+11.7
12	6 50	33.85	53.25	53.0	53.1	53.46	+ 61.8	+ 8.6	+ 5.4	—53.2	— 56.6	+ 6.8	+ 7.0
12	8 00	54.0	53.95	53.8	53.7	53.40	+ 61.4	+ 8.8	+ 0.6	—52.6	— 60.8	+ 6.4	— 3.4
12	9 10	53.6	53.6	53.05	3.0	53.38	+ 61.8	+ 6.1	— 1.8	—55.7	— 63.6	+ 6.0	— 3.1
12	10 30	55.5	55.85	56.0	56.0	55.45	+ 56.4	+ 7.1	+ 0.7	—49.3	— 55.7	— 0.4	—15.2
12	11 15	55.9	56.1	56.0	56.0	56.12	+ 57.9	+ 5.9	+ 2.7	—52.0	— 55.2	— 5.9	—16.9
12	11 40	56.15	56.45	56.3	56.4	56.56	+ 58.0	+ 4.9	+ 5.1	—53.1	— 52.9	— 8.6	—17.7
12	11 55	Removed R_2 R_3 and G to open air exposure											

At 1^h 35^m, temperature by No. 1, air contact =44°.5 and by No. 2 by contact with R_2=45°.7.
Removed G. R_2 and R_3 to comparator.

EXHIBIT L.

Continuation of the comparisons between meters R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	$R_2—R_3$	$R_2—G_1$	$R_2—R_3$	$R_2—G_1$
Oct.	h m												
12	2 15	53°.0	53°.55	....	....	.....	+104.5 div.	+41.7 div.	+ 40.3 div.	—62.8 div.	— 64.2 div.	+ 3.0 div.	— 3.3 div.
12	2 30	54.7	54.8	....	....	.....	+ 97.2	+33.1	+ 25.9	—64.1	— 73.1	— 9.8	—21.4
12	2 45	54.7	55.1	....	....	56°.62	+ 79.6	+34.0	+ 20.3	—45.6	— 59.3	+ 8.7	—12.1
12	3 00	55.1	55.7	....	....	56.66	+ 73.5	+18.1	+ 17.1	—55.4	— 56.4	— 3.8	—14.5
12	3 15	55.6	55.9	....	....	56.66	+ 62.9	+12.6	+ 14.2	—50.3	— 48.7	— 2.1	— 8.6
12	3 35	55.7	55.9	...	....	56.66	+ 59.4	+ 8.7	+ 13.4	—50.7	— 46.0	— 3.2	— 5.9
12	2 30	55.8	55.75	....	....	56.34	+ 54.0	+ 4.0	+ 14.7	—50.0	— 39.3	— 3.2	+ 2.1
12	5 10	55.6	55.4	....	....	56.05	+ 53.0	+ 8.5	+ 10.3	—44.5	— 42.7	+ 3.7	+ 1.8
13	10 15	53.9	53.95	53°.9	53°.8	34.00	+ 65.0	+11.1	+ 12.5	+53.9	— 52.5	+ 5.8	+ 4.9
12	11 00	53.8	53.5	52.8	53.0	53.67	+ 63.9	+ 9.6	+ 9.8	—54.3	— 54.1	+ 6.1	+ 7.3
13	11 30	53.85	53 5	52.8	53.0	53.62	+ 63.9	+ 3.3	+ 8.8	—60.6	— 55.1	— 0.6	+ 6.3
13	12 00	53.7	53.55	53.0	53.05	53.60	+ 61.5	+ 1.9	+ 5.9	—59.6	— 55.6	+ 1.5	+ 5.3
13	12 30	54.0	54.0	53.9	54.0	53.67	+ 62.3	+ 2.5	+ 5.0	—59.8	— 57.3	— 0.8	— 0.4
13	1 30	53.13	54.85	55.0	54.85	54.32	+ 61.1	+ 4.8	+ 0.6	—56.3	— 60.5	— 4.9	—11.1
13	2 00	57.0	57.1	56.1	55.85	55.99	+ 49.9	+ 3.1	— 1.9	—46.8	— 51.8	— 8.1	—22.3
13	2 20	——Opened both windows. Closed windows at 2h 45m Outside air =54°.0.——											
13	3 00	56.1	56.7	57.0	57.0	.....	+ 54.2	+ 4.8	— 1.1	—49.4	— 55.3	— 4.6	—22.3
14	——Both windows open during the night of Oct. 13th. Closed at 6h 15m Oct. 14th.——												
14	6 10	43.9	43.5	41.8	41.8	43.88	+ 12.77	+ 4.9	— 7.7	—122.8	—135.4	+ 4.7	—14.7
14	6 40	45.0	45.0	44.3	44.1	43.79	+ 12.68	+ 4.7	— 14.6	- 122.1	—141.4	— 1.5	— 4.6
41	7 40	46.7	47.5	48.2	48.1	.....	+ 12.24	+ 6.3	— 18.0	—116.1	—140.4	— 7.6	—25.8

EXHIBIT L.

Continuation of the comparisons between meters R_2, R_3 and G_1.

The reductions are made from the readings of thermometers Nos. 1 and 2 with the relative coefficient 6.78 div. for R_2—R_3 and 8.87 div. for R_2—G_1.

Date	Time	No 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1	R_2—R_3	R_2—G_1
	h m	°	°	°	°	°							
14	9 20	49.4	50.4	52.0	51.9	48.70	+106.8 div.	+ 6.7 div.	— 25.4 div.	—100.1 div.	—132.2 div.	—11.9 div.	—35.0 div.
14	10 00	49.9	51.0	53.0	53.0	48.82	+103.8	+ 9.7	— 18.6	— 93.1	—121.4	— 6.2	—37.8
14	10 10	sky overcast.											
14	12 30	55.0	55.4	53.85	52.95	53.06	+ 83.2	+ 8.6	— 7.1	— 74.6	— 90.3	—22.3	—45.8
14	2 15	54.6	54.9	56.2	56.05	54.48	+ 73.3	+ 6.4	— 3.2	— 66.9	— 76.5	—11.9	—27.6
14	3 40	54.95	55.5	56.2	56.2	55.26	+ 64.1	+ 6.2	— 2.6	— 57.9	— 66.7	— 5.3	—23.1
14	4 50	55.45	55.8	56.25	56.2	55.70	+ 58.3	+ 6.4	— 1.5	— 51.9	— 59.8	— 2.7	—18.8
16	6 10	45.9	45.65	44.8	44.8	45.86	+112.7	+ 2.5	— 4.7	—110.2	—117.4	+ 3.7	+13.6
16	6 40	46.1	45.85	45.1	45.2	45.73	+114.3	+ 4.1	— 4.3	—110.2	—118.6	+ 2.4	+10.6
16	7 20	46.0	46.0	45.5	45.5	45.82	+116.6	+ 3.7	— 5.1	—112.9	—121.7	+ 0.4	+ 6.2
16	1 40	46.35	46.35	46.0	46.1	45.95	—117.9	+ 4.9	— 8.6	—113.0	—126.5	— 2.1	— 1.7
16	9 30	47.00	47.00	47.0	47.05	46.33	+115.3	+ 7.4	— 11.6	—107.9	—127.2	— 1.4	— 8.2
16	10 00	47.75	47.95	48.5	48.4	46.99	+114.1	+ 5.1	— 15.3	—109.9	—129.4	— 8.5	—18.8
16	11 00	49.0	49.9	51.3	51.1	48.65	+105.8	+ 5.0	— 12.9	—100 8	—118.9	— 7.9	—25.6
16	12 00	50.25	51.2	53.0	52.9	50.36	+ 99.8	+ 5.9	— 14.8	— 93.9	—114.6	— 9.5	—32.8
16	1 50	52.1	52.9	54.3	54.2	53.20	+ 85.5	+ 6.5	— 8.8	— 79.0	— 94.3	— 7.1	—27.6
16	3 45	53.5	54.0	54.95	55.05	53.74	+ 76.6	+ 7.4	— 0.8	— 69.2	— 77.4	— 6.8	—20.4
16	5 10	54.0	54.5	55.1	55.0	54.32	+ 68.7	+ 3.0	+ 4.1	— 65 7	— 64.6	— 6.7	—12.1
17	6 20	51.25	51.15	54.5	50.9	51.31	+ 83.6	+ 5.1	+ 10.4	— 78.5	— 73 2	— 0 9	+ 9.2
17	6 50	51.05	51.00	50.5	50.8	51.26	+ 83.8	+ 6.5	+ 9.2	— 77.3	— 74.6	+ 1.7	+ 9.0

EXHIBIT L.

Continuation of the comparisons between meters R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1	R_2—R_3	R_2—G_1
Oct.	h m	°	°	°	°	°							
17	7 50	51.05	50.95	50.7	50.8	50.89	+ 83.5 div.	..... div.	+ 8.2 div.	...... div.	— 75.2 div.	......	+ 8.8 div.
17	9 20	51.20	51.25	51.2	51.2	50.93	+ 82.8	.....	+ 4.5	......	— 78.3	......	+ 3.0
18	6 50	55.6	55.8	55.8	55.9	55.27	— 3.7	—57.3	— 55.6	— 53.6	— 51.9	— 5.4 div.	—10.9
18	7 20	55.5	55.6	55.6	55.9	55.42	— 5.6	—60.0	— 53.9	— 54.4	— 48.3	— 5.5	— 5.5
25	12 30	55.9	.....	56.3	....	55.31	+ 8.4	+58.9	......	— 50.5	......	— 4.3	
25	4 45	55.45	.....	55.0	....	55.80	+ 18.0	+71.4	......	— 53.4	......	— 4.2	
25	9 00	53.2	.....	53.0	....	54.05	+ 30.9	+92.8	......	— 61.9	......	+ 2.6	
27	6 10	46.95	.....	46.95	....	46.85	+140.9	+32.8	......	—108.1	......	— 1.3	
	6 20	46.95	.....	46.95	....	46.82	+140.0	+32.0	......	—108.0	......	— 1.2	
	6 30	46.9	.....	46.9	....	46.80	+137.1	+30.3	......	—106.8	......	+ 0.4	
	6 40	46.9	.....	46.9	....	46.78	+139.1	+29.6	......	—109.5	......	— 2.3	
	6 50	46.9	.....	46.8	....	46.77	+140.5	+31.2	......	—109.3	......	— 2.1	
	7 00	46.8	.....	46.8	....	46.76	+139.9	+32.1	......	—107.8	......	+ 0 0	
	7 10	46.8	.....	46.8	....	46.76	+139.3	+29.1	......	—110.2	......	— 2.4	
	7 20	46.8	.....	46.8	....	46.75	+139.4	+30.9	......	—108.5	......	— 0.7	
	8 15	46.05	.....	45.95	....	46.69	+140.4	+28.2	......	—112.3	......	+ 0.6	
	8 20	46.1	.....	46.0	....	46.68	+140.0	+27.0	......	—113.0	......	— 0.4	
	8 25	46.1	.....	46.0	....	46 65	+140.8	+28.4	......	—112.4	......	+ 0.2	
	8 50	46.3	.....	46.15	....	46.63	+141.6	+25.9	......	—115.7	......	— 4.5	
	8 35	46.3	.....	46.15	....	46.63	+140.7	+23.4	......	—117.3	......	— 6.1	
	9 30	46.6	.....	46.3	....	46.71	+136.5	+23.3	......	—113.2	......	— 4.0	
	9 35	46.6	.....	46.3	....	46.71	+135.6	+21.5	......	—114.1	......	— 4.9	

EXHIBIT L.

Continuation of the comparisons between meters R_2, R_3 and G_1.

Date Nov.	Time h m	No. 1	No. 2	No. 3	No. 4	No ·5	R_2	R_3	G_1	R_2—R_3	R_2—G_a
		°	°	°		°					
3	6 30	59.9	58.9	58.3	....	61.39	+ 24.4 div.	+18.0 div.	+51.1 div.	+ 12.6 div.	+40.2 div.
3	6 55	57.8	57.8	57.0	....	60.89	+ 34.3	+19.1	+47.8	+ 11.3	+36.7
3	7 10	57.9	57.9	57.0	....	60.51	+ 38.0	+20.5	+48.4	+ 13.1	+39.0
3	8 05	58.0	57.0	56.7	....	59.47	+ 45.1	+18.8	+45.0	+ 5.6	+30.1
3	9 10	57.9	57.2	57.5	....	58.62	+ 45.1	+15.4	+38.7	+ 2.9	+22.2
3	9 35	57.9	57.7	58.0	....	58.64	+ 46.7	+13.0	+30.6	— 0.1	+ 7.6
3	10 00	58.3	58.05	58.1	....	58.64	+ 43.1	+ 9.9	+24.2	— 3.3	+ 1.5
3	10 25	58.1	58.1	58.1	....	58.62	+ 41.7	+ 9.2	+22.9	— 1.3	+ 1.8
3	10 40	58.2	58.2	58.2	....	58.60	+ 42.4	+ 8.3	+23.6	— 3.5	+ 0.9
3	10 55	58.4	58.4	58.6	....	58.58	+ 42.0	+ 9.7	+24.5	— 3.1	+ 0.5
3	11 10	58.5	58 4	58.4	....	58.62	+ 41.3	+ 8.9	+20.8	— 3.9	— 2.5
3	11 30	58.6	58.8	58.7	....	58.68	+ 36.7	+ 5.5	+16.9	— 3.4	— 5.4
3	11 50	58.7	58.9	58.7	....	58.70	+ 37.6	+ 6.3	+20.0	— 4.1	— 4.1
3	12 10	58.9	59.0	59.0	....	58.72	+ 38.2	+ 4.2	+20.2	— 8.2	— 5.4
3	12 13	———Opened windows in the partition between rooms. Temperature in adjoining room =98°———									
3	12 20	60.4	61.2	61.3	....	59.25	+ 33.8	+ 7.4	+ 8.7	— 10.5	—32.1
3	1 15	63.2	63.4	64.2	....	60.22	+ 20.1	— 1.0	— 1.1	— 21.7	—50.3
3	1 35	64.5	64.9	65.6	....	60.58	+ 12.0	— 3.8	— 4.8	— 27.9	—56.5
3	1 50	64.9	66.3	66.3	....	61.21	+ 8.8	— 6.3	— 5.5	— 30.6	—67.3
3	5 00	62.9	62.9	62.7	....	62.33	+ 2.2	— 0.9	......	— 6.3	.....
3	5 45	63.0	62.9	62.9	....	62.53	+ 3.5	+ 0.8	......	— 5.9	.....
3	6 40	61.8	61.7	61.2	....	62.06	+ 5.8	+ 2.2	......	+ 3.9	.....

EXHIBIT L.

Continuation of the comparisons between meters R_2, R_3 and G_1.

Date	Time	No. 1	No. 2	No. 3	No. 4	No. 5	R_2	R_3	G_1	R_2—R_3	R_2—G_1
Nov.	h m	°	°	°		°					
3	7 20	61.1	61.0	61.0	....	61.70	+ 10.9 div.	+ 6.3 div.	......	+ 9.1 div.	.....
5	9 50	59.1	58.9	58.9	....	60.04	+ 24.8	+ 6.3	......	+ 13.8	.....
3	10 05	58.8	58.6	58.8	....	59.92	+ 28.3	+ 6.8	......	+ 13.5	.....
3	10 50	58.7	58.2	58.1	....	59.50	+ 33.6	+ 9.0	......	+ 4.4	.....
3	11 30	58.7	58.1	58.1	....	59.13	+ 36.0	+ 7.4	......	— 0.1	.....
2	12 00	58.4	57.9	58.6	....	58.78	+ 36.3	+ 6.9	......	— 0.2	.....
3	12 30	58.9	58.7	58.6	....	58.95	+ 36.2	+ 5.9	......	+ 3.8	.....

EXHIBIT M.

Experiment 1.

Change in length of R_2 in passing from 34°.05 to 64°.1.
Removal to comparator at 7^h 10^m.

The values of T in the second column relate to the temperatures of the bar of metal after removal to the comparator. The values in the third column relate to the air temperature of the comparing room.

Time	T	T	Div.						
h m	°	°							
7 35	....	64.5	+218.2*						
7 51	58.1	64.3	+150.7						
				—160.9					
8 7	59.0	64.5	+100.3						
8 23	59.4	64.3	+ 57.3*		+68.7				
8 39	59.5	64.2	+ 19.1						
				— 92.2		—30.3			
8 55	59.5	64.4	— 10.7						
9 11	59.4	64.5	— 34.9*		+38.4		+ 8.9		
9 27	60.9	64.6	— 54.7						
				— 53.8		—21.4		+10.4	
9 43	6.15	64.5	— 72.1						
9 59	62.1	64.8	— 88.7*.........		+17.0........		+19.3........		—25.6
10 15	63.0	64.8	—102.9*						
				— 36.8		— 2.1		— 15.2	
10 31	63.6	64.7	—116.5						
10 47	64.1	64.7	—125.5*		+14.9		+ 4.1		
11 3	64.0	64.7	—132.7						
				— 21.9		+ 2.0			
11 41	63.6	64.0	—142.8						
11 57	63.6	64.0	—147.4*		+16.9				
12 13	....	....	....						
				— 5.0					
12 29	....	....	....						
12 45	64.1	64.1	—152.4*						

EXHIBIT M.— *Continued.*

Experiment 2.

Change in length of R_2 in passing from $35^\circ.05$ to $50^\circ.2$.
Removal to comparator at $9^h\ 43^m$.

Time	T	T	Div.						
h m	°	°							
10 0	44.2	49.2	+159.7*						
10 16	43.6	49.4	+115.2						
				—106.9					
10 32	44.6	49.5	+ 80.2						
10 48	45.7	49.4	+ 52.8*		+54.6				
11 4	46.4	49.6	+ 31.8						
				— 52.3		—30.1			
11 20	47.2	49.7	+ 14.6						
11 36	47.6	49.7	+ 0.5*		+24.5		+17.9		
11 52	48.0	49.7	— 8.9						
				— 27.8		—12.2		— 8.9	
11 8	48.2	49.2	— 18.0						
12 24	48.8	49.7	— 27.3*	....	+12.3		+ 9.0		+ 0.4
12 40	48.9	49.7	— 31.0						
				— 15.5		— 3.2		— 8.5	
12 58	49.2	49.8	— 38.2						
1 14	49.5	50.1	— 42.8*		+ 9.1		+ 0.5		
1 30	49.8	50.1	— 47.4						
				— 6.4		— 2.7			
1 45	49.9	50.0	— 49.5						
2 15	49.5	49.4	— 49.2*		+ 6.4				
2 31	....	....	....						
				— 0.0					
2 47	....	....	....						
3 03	50.1	50.1	— 49.2*						

EXHIBIT M.— *Continued.*

Experiment 3.

Change in length of R_2 in passing from 59°.05 to 56°.8.
Removal to comparator at $1^h\ 50^m$.

Time	T	T	Div.						
h m	°	°							
2 15	57.7	56.2	+ 12.3*						
2 31	57.7	56.3	20.7	+ 12.9					
2 47	57.5	56.7	25.2*		— 2.3				
3 3	57.3	56.7	30.1	+ 10.6		— 0.9			
3 19	57.2	56.7	35.8*		— 3.2		— 1.0		
3 35	57.1	56.8	39.8	+ 7.4		— 1.9		— 7.9	
3 51	57.0	56.7	43.2*		— 5.1		+ 6.9		—21.6
4 7	57.0	56.8	44.7	+ 2.3		+ 5.0		—13.7	
4 23	56.9	56.7	45.5*		— 0.1		— 6.8		
4 39	56.9	56.7	47.3	+ 2.2		— 1.8			
4 55	57.0	56.7	47.7*		— 1.9				
5 11	57.0	56.8	48.0	+ 0.3					
5 27	56.8	56.6	+48.0*						

Experiment 4.

Change in length of R_2 in passing from 68°.0 to 64°.1.
Removal to comparator at $2^h\ 26^m$.

h m	°	°							
2 41	65.1	64.6	—201.7*						
2 57	66.0	65.0	—188.8	+ 23.9					
3 13	65.1	65.1	—177.8*		—13.9				
3 29	64.6	65.0	—171.0	+ 10.0		+ 6.5			
3 45	64.4	64.8	—167.8*		— 7.4		+ 0.5		
4 1				+ 2.6		+ 7.0		—7.7	
4 17	64.2	64.4	—165.2		— 0.4		— 7.2		+12.6
4 23				+ 2.2		— 0.2		+4.9	
4 49	64.3	64.9	—163.0*		— 0.6		— 2.3		
5 5				+ 1.6		— 2.5			
5 21	64.7	64.6	—161.4		— 3.1				
5 37				— 1.5					
5 53	64.1	63.9	—162.9*						

EXHIBIT M.— *Continued.*

Experiment 5.

Change in length of R_3 in passing from 34°.05 to 64°.1.
Removal to comparator at 7^h 10^m.

Time h m	T °	T °	Div						
7 39	....	64.5	+ 64.2*						
7 55	58.1	64.3	+ 21.3						
				—100.9					
8 11	56.1	64.5	— 10.3						
8 27	56.1	64.2	— 36.7*		+48.8				
8 43	58.5	64.3	— 58.9						
				— 52.1		—24.4			
8 59	59.0	64.5	— 77.3						
9 15	59.4	64.7	— 88.8*		+24.4		+10.1		
9 31	60.9	64.7	— 98.7						
				— 27.7		—14.3		+ 1.8	
9 47	62.1	64.5	—107.4						
10 3	62.5	64.8	—116.5*	....	+10.1	......	+11.9	......	—10.1
10 19	62.6	64.7	—122.4						
				— 17.6		— 2.4		— 8.3	
10 36	63.0	64.6	—129.9						
10 51	64.0	64.8	—134.1*		+ 7.7		+ 3.6		
11 7	64.2	64.9	—140.4						
				— 9.9		+ 1.2			
11 23	64.1	64.7	—142.3						
11 39	64.1	64.7	—144.0*		+ 8.9				
11 55		....							
				— 1.0					
12 11		....							
12 27	64.1	64.7	145.0*						

EXHIBIT M.— *Continued.*

Experiment 6.

Change in length of R_3 in passing from 32°.05 to 49°.7.
Removal to comparator at 9^h 43^m.

Time h m	T °	T °	Div.						
9 50	44.0	49.2	+ 6.2*						
10 6	43.3	49.2	— 34.8						
				— 68.7					
10 22	44.0	49.9	— 56.0						
10 38	44.8	49.4	— 74.9*		+33.6				
10 54	46.1	49.5	— 91.2						
				— 35.1		—15.2			
11 10	46.9	49.5	—103.0						
11 26	47.4	49.7	—110.0*		+18.4		+ 3.8		
11 42	47.8	49.7	—115.0						
				— 16.7		—11.4		+ 5.1	
11 58	48.1	49.5	—120.7						
12 14	48.5	49.4	—126.7*	....	+ 7.0	......	+ 8.9	......	—11.8
12 30	48.9	49.8	—130.4						
				— 9.7		— 2.5		— 6.7	
12 46	49.1	49.7	—133.5						
1 2	49.4	49.0	—136.4		+ 4.5		+ 2.2		
1 18	49.8	50.0	—139.6						
				— 5.2		— 0.3			
1 34	49.9	50.0	—142.7						
2 20	49.7	49.6	—141.6*		+ 4.2				
2 36	....	....	....						
				— 1.0					
2 52	....	....	....						
3 08	49.9	49.6	142.6						

EXHIBIT M.— *Continued.*

Experiment 7.

Change in length of R_3 in passing from 59°.05 to 56°.6.
Removal to comparator at 1^h 59^m.

Time		T	T	Div.						
h	m	°	°							
2	8	....	56.1	— 27.6*						
2	24	57.7	56.2	— 19.5	+ 12.6					
2	40	57.6	56.4	— 15.0*		— 6.1				
2	56	57.6	56.6	— 11.3	+ 6.5		+ 5.7			
3	12	57.2	56.7	— 8.5*		— 0.4		— 7.5		
3	28	57.2	56.7	— 5.8	+ 6.1		— 1.8		+ 7.9	
3	44	57.1	59.8	— 2.4*........		— 2.2........		+ 0.4........		— 1.7
4	0	57.0	56.8	— 0.8	+ 3.9		— 1.4		+ 6.2	
4	16	56.9	56.8	+ 1.5*		— 3.6		+ 5.8		
4	32	56.9	56.7	+ 0.6	+ 0.3		+ 4.4			
4	48	56.9	56.7	+ 1.8*		+ 0.8				
5	4	56.8	56.7	+ 2.9	+ 1.1					
5	20	56.6	56.5	+ 2.9*						

Experiment 8.

Change in the length of R_3 in passing from 68°.0 to 64°.1.
Removal to comparator at 2^h 26^m.

h	m	°	°							
2	49	65.1	64.6	—175.7*						
3	1	65.7	65.2	—170.6	+ 10.3					
3	17	65.1	65.2	—165.4*		— 5.1				
3	33	64.8	64.9	—162.1	+ 5.2		+ 1.7			
3	49	64.4	64.8	—160.2*		— 3.4		+ 1.8		
4	5				+ 1.8		+ 3.5		— 7.0	
4	21	64.2	64.3	—158.4*........		+ 0.1........		— 5.2........		+14.3
4	37				+ 1.9		— 1.7		+ 7.3	
4	53	64.6	64.7	—156.5		— 1.6		+ 2.1		
5	9				+ 0.3		+ 0.4			
5	25	64.7	64.6	—156.2*		— 1.2				
					— 0.9					
5	58	61.1	63.9	—157.1*						

EXHIBIT M.— *Continued.*

Experiment 9.

Change in length of G in passing from 34°.05 to 63°.6.
Removal to comparator at 7^h 10^m.

Time	T	T	Div.						
h m	°	°							
7 31	....	....	+ 15.6*						
7 47	58.2	64.5	− 29.0						
				− 92.5					
8 3	58.6	64.4	− 58.5						
8 17		64.3	− 76.9*		+56.7				
8 35	60.2	64.2	− 91.8						
				− 35.8		−34.9			
8 51	60.1	64.5	−103.6						
9 7	61.9	64.5	−112.7*		+21.8		+22.2		
9 23	62.5	64.8	−117.8						
				− 14.0		−12.7		−15.9	
9 39	63.2	64.7	−122.6						
9 55	63.4	64.8	−126.7*		+ 9.1		+ 6.3		+15.2
10 11	63.7	64.7	−127.9						
				− 4.9		− 6.4		− 0.7	
10 27	63.6	64.7	−129.4						
10 43	63.5	64.3	−131.6		+ 2.7		+ 5.6		
10 79	63.8	64.8	−133.5						
				− 2.2		− 0.8			
11 15	63.5	64.7	−133.6						
12 10	63.6	64.0	−133.8*		+ 1.9				
12 45	63.6	63.7	−134.0						
				− 0.3					
1 01	63.6	63.7	−135.0						
1 17	63.6	63.7	−134.1*						

EXHIBIT M.— *Continued.*

Experiment 10.

Change in length of G. in passing from 35°.5 to 50°.0.
Removal to comparator at 7^h 10^m.

Time h m	T °	T °	Div.					
2 36	....	49.9	— 60.3*					
2 52	46.8	50.1	— 90.9	—50.1				
3 8	48.0	49.9	—110.4*		+26.5			
2 22	48.6	50.0	—124.8	—23.6		—11.3		
3 38	48.7	49.8	—134.0*		+15.2		+ 0.9	
3 56	49.0	49.9	—138.5	— 8.4........		—12.4.........		+12.3
4 10	49.1	49.7	—142.3*		+ 2.8		+13.2	
4 26	49.5	49.7	—148.3	— 5.6		+ 0.8		
4 42	49.7	49.9	—147.6*		+ 3.6			
4 58	49.9	50.0	—149.5	— 3.0				
5 14	50.1	50.1	—150.6*					

Experiment 11.

Change in length of G in passing from 86°.0 to 63°.0.
Removal to comparator at 2^h 26^m.

h m	°	°							
2 37	66.0	64.6	—157.9*						
1 53	46.3	65.0	—152.8	+ 10.8					
3 9	65.4	64.9	—147.1*		— 3.4				
3 25	64.7	65.1	—144.5	+ 7.4		+ 0.3			
3 41	64.9	64.9	—139.7*		— 3.1		— 0.6		
3 57	....	....	....	+ 4.3		— 0.3		+ 2.9	
4 13	64.2	64.4	+135.4*........		— 3.4........		+ 2.3........		— 4.8
4 28	....	....	....	+ 0.9		+ 2.0		— 1.9	
4 45	64.8	65.1	—135.4*		— 1.4		+ 0.4		
5 1	....	....	....	— 0.5		+ 2.4			
5 17	64.4	64.4	—135.0*		+ 1.0				
5 33	....	....	....	+ 0.5					
6 50	63.0	62.3	—131.0*						

EXHIBIT M.— *Continued.*

Experiment 12.

Change in length of G in passing from melting ice to 65°.0
Removal to comparator at $9^h\ 42^m$.

Time	Div.					
h m						
9 56	+ 93.89*					
10 12	148.95	+96.59				
10 28	190.48*		—56.19			
10 44	214.76	+40.40		+36.63		
11 0	230.88*		—19.56		—26.61	
11 16	241.90	+20.84		+10.02		+18.89
11 32	251.72*		— 9.54........		— 7.72........	—8.86
11 48	259.30	+11.30		+ 2.30		+10.03
12 4	263.02*		— 7.24		+ 2.31	
12 29	264.78	+ 4.06		+ 4.61		
12 36	267.08*		— 2.63			
12 52	268.32	+ 1.43				
1 8	268.51*					

OMISSION.—Insert the following after division (17), page 74:

The standards compared must remain in the same position with respect to each other, and with respect to the base upon which they rest throughout the entire series of observations undertaken.

If the adjacent vertical faces of the standards are in actual contact, the amount by which the change in length fails to follow the indications of the thermometers will not be the same as when the standards are separated by an interval of e. g. one centimeter. In other words, the character of the systematic relations between the residuals will undergo a change in passing from contact to independent positions. One bar seems to hold the other in check during variations of temperature. In a general way the surface indications seem to give more correct results when the bars are in contact, and the indications of the thermometer at the center of the standard bar seem to be more reliable when the standards are placed upon blocks of wood and separated one or two centimeters.